7.7 Content for Stage 5

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Mathematics • Stage 5 (5.1 pathway)

Number and Algebra
Financial Mathematics

Outcomes
A student:

• uses appropriate terminology, diagrams and symbols in mathematical contexts	MA5.1-1WM
• selects and uses appropriate strategies to solve problems	MA5.1-2WM
• provides reasoning to support conclusions which are appropriate to the context	MA5.1-3WM
• solves financial problems involving earning, spending and investing money	MA5.1-4NA

Related Life Skills outcomes: *MALS-1WM, MALS-2WM, MALS-3WM, MALS-11NA, MALS-12NA, MALS-13NA, MALS-14NA*

Students:

Solve problems involving earning money

- calculate wages given an hourly rate of pay, including overtime and special rates for Sundays and public holidays [N, WE, PSC]
  - ‣ use classifieds and online advertisements to compare pay rates and conditions for different positions (Problem Solving) [N, WE, PSC]
  - ‣ read and interpret examples of pay slips (Communicating) [N, WE, PSC]
- calculate earnings for various time periods from non-wage sources including salary, commission and piecework [N, WE, PSC]
- calculate leave loading as 17.5% of normal pay for up to four weeks [N, WE, PSC]
  - ‣ research the reasons for inclusion of leave loading provisions in many awards (Reasoning) [N, WE]
- determine the weekly, fortnightly or monthly tax to be deducted from a worker's pay [N, WE, PSC]
- determine taxable income by subtracting allowable deductions and use current rates to calculate the amount of tax payable for the financial year [N, WE, PSC]
  - ‣ determine a worker's tax refund or liability by comparing the tax payable for a financial year with the tax already paid under the Australian PAYG system (Problem Solving) [N, WE, PSC]
  - ‣ investigate how rebates and levies, including the Medicare levy and Family Tax Benefit, affect different workers' taxable incomes (Problem Solving) [N, WE]

Solve problems involving simple interest (ACMNA211)

- calculate simple interest using the formula $I = PRN$, where I is the interest, P is the principal, R is the interest rate per time period (expressed as a fraction or decimal) and N is the number of time periods

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Number and Algebra
Financial Mathematics

- apply the simple interest formula to problems related to investing money at simple interest rates [N]
  - find the total value of a simple interest investment after a given time period (Problem Solving) [N]
  - calculate the principal or time needed to earn a particular amount of interest given the simple interest rate (Problem Solving) [N]
- calculate the cost of buying expensive items by paying an initial deposit and making regular repayments that include simple interest [N, PSC]
  - investigate other fees and charges related to 'buy today, no more to pay until …' promotions (Problem Solving) [N, CCT, PSC]
  - compare total cost of buying on terms to paying cash (Problem Solving) [N, PSC]
  - recognise that repossession does not remove financial debt (Reasoning) [N, PSC]

Connect the compound interest formula to repeated applications of simple interest using appropriate digital technologies (ACMNA229)

- calculate compound interest for two or three years using repetition of the formula for simple interest
  - connect the calculation of the total value of a compound interest investment to repeated multiplication using a calculator, eg a rate of 5% per annum leads to repeated multiplication by 1.05 (Communicating) [N, ICT]
  - compare simple interest with compound interest in practical situations, eg to determine the most beneficial investment or loan (Communicating, Reasoning) [N, PSC]
  - compare simple interest with compound interest on an investment over various time periods using tables, graphs or spreadsheets (Reasoning) [L, N, ICT, PSC]

Background information

Pay-as-you-go (PAYG) is the Australian taxation system for withholding taxation from employees in their regular payments from employers. The appropriate level of taxes are withheld from an employee's payment and passed on to the Australian Taxation Office. Deduction amounts will reduce the taxation debt that may be payable following submission of a tax return, or alternatively will be part of the refund given for overpayment.

Simple interest is commonly used for short-term investments or loans. As such, time periods may be given in months or even days. Students should be encouraged to convert the interest rate per period to a decimal, eg 6% per annum = 0.005 per month, which is then used as R in the formula.

It is not intended at this stage for students to use the formula for compound interest.

Internet sites may be used to find commercial rates for home loans and 'home loan calculators'.

Mathematics • Stage 5 (5.1 pathway)

Number and Algebra
Indices

Outcomes
A student:

- uses appropriate terminology, diagrams and symbols in mathematical contexts — MA5.1-1WM
- provides reasoning to support conclusions which are appropriate to the context — MA5.1-3WM
- operates with algebraic expressions involving positive-integer and zero indices, and establishes the meaning of negative indices for numerical bases — MA5.1-5NA

Students:

Extend and apply the index laws to variables, using positive integer indices and the zero index (ACMNA212)

- use the index laws previously established for numbers to develop the index laws in algebraic form,
  eg $2^2 \times 2^3 = 2^{2+3} = 2^5 \quad a^m \times a^n = a^{m+n}$
  $2^5 \div 2^2 = 2^{5-2} = 2^3 \quad a^m \div a^n = a^{m-n}$
  $\left(2^2\right)^3 = 2^6 \quad \left(a^m\right)^n = a^{mn}$
  - ‣ explain why a particular algebraic sentence is incorrect, eg explain why $a^3 \times a^2 = a^6$ is incorrect (Communicating, Reasoning) [CCT]
- establish that $a^0 = 1$ using the index laws [N],
  eg $a^3 \div a^3 = a^{3-3} = a^0$
  and $a^3 \div a^3 = 1$
  $\therefore \quad a^0 = 1$
  - ‣ explain why $x^0 = 1$ (Reasoning) [CCT]
- simplify expressions that involve the zero index, eg $5x^0 + 3 = 8$

Simplify algebraic products and quotients using index laws (ACMNA231)

- simplify expressions that involve the product and quotient of simple algebraic terms containing indices, eg $\left(3x^2\right)^3 = 27x^6$, $2x^2 \times 3x^3 = 6x^5$, $15a^6 \div 3a^2 = 5a^4$, $\dfrac{3a^2}{15a^6} = \dfrac{1}{5a^4}$
  - ‣ compare expressions such as $3a^2 \times 5a$ and $3a^2 + 5a$ by substituting values for a (Communicating, Reasoning) [CCT]

Apply index laws to numerical expressions with integer indices (ACMNA209)

- establish the meaning of negative indices, eg by patterns [N]

3^3	3^2	3^1	3^0	3^{-1}	3^{-2}
27	9	3	1	$\frac{1}{3}$	$\frac{1}{3^2} = \frac{1}{9}$

- evaluate numerical expressions with a negative index by first rewriting with a positive index, eg $3^{-4} = \dfrac{1}{3^4} = \dfrac{1}{81}$
- translate numbers to index form (integer indices only) and vice versa

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Number and Algebra
Linear Relationships

Outcomes
A student:

- uses appropriate terminology, diagrams and symbols in mathematical contexts MA5.1-1WM
- provides reasoning to support conclusions which are appropriate to the context MA5.1-3WM
- determines the midpoint, gradient and length of an interval, and graphs linear relationships MA5.1-6NA

Related Life Skills outcomes: *MALS-1WM, MALS-3WM, MALS-29MG, MALS-30MG, MALS-31MG*

Students:

Find the midpoint and gradient of a line segment (interval) on the Cartesian plane using a range of strategies, including graphing software (ACMNA294)

- determine the midpoint of an interval using a diagram [N]
- use the process for calculating the 'mean' to find the midpoint, M, of the interval joining two points on the Cartesian plane [N]
  - ‣ explain how the concept of average is used to calculate the midpoint of an interval (Communicating) [L, CCT]
- plot two points to form an interval on the Cartesian plane and to form a right-angled triangle by drawing a vertical side from the higher point and a horizontal side from the lower point
- use the interval between two points on the Cartesian plane as the hypotenuse of a right-angled triangle and the relationship $\text{gradient} = \frac{\text{rise}}{\text{run}}$ to find the gradient of the interval joining two points [N]
  - ‣ describe the meaning of the gradient of a line joining two points and explain how it can be found (Communicating) [L, CCT]
  - ‣ distinguish between positive and negative gradients from a diagram (Reasoning) [CCT]
- use graphing software to find the midpoint and gradient of an interval [ICT]

Find the distance between two points located on a Cartesian plane using a range of strategies, including graphing software (ACMNA214)

- use the interval between two points on the Cartesian plane as the hypotenuse of a right-angled triangle and Pythagoras' theorem to determine the length of the interval joining the two points [N]
  - ‣ describe how the length of an interval joining two points can be calculated using Pythagoras' theorem (Communicating) [L, N]
- use graphing software to find the distance between two points on the Cartesian plane [ICT]

Sketch linear graphs using the coordinates of two points and solve linear equations (ACMNA215)

- construct tables of values and use coordinates to graph vertical and horizontal lines such as $x = 3$, $x = -1$, $y = 2$, $y = -3$ [N, CCT]
- identify the x- and y-intercepts of lines [N]
- identify the x-axis as the line $y = 0$ and the y-axis as the line $x = 0$ [L, N]
  - ‣ explain why the x- and y-axes have these equations (Communicating, Reasoning) [L]

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Number and Algebra
Linear Relationships

- graph a variety of linear relationships on the Cartesian plane with and without ICT
  eg $y = 3 - x$, $y = \frac{x+1}{2}$, $x + y = 5$, $x - y = 2$, $y = \frac{2}{3}x$ [N, ICT, CCT]
  - compare and contrast equations of lines that have a negative gradient and equations of lines that have a positive gradient (Communicating, Reasoning) [CCT]
- determine whether a point lies on a line by substitution

Solve problems involving parallel and perpendicular lines (ACMNA238)

- determine that parallel lines have equal gradients
  - use ICT to compare graphs of a variety of straight lines with their respective gradients and establish the condition for lines to be parallel (Communicating, Reasoning) [ICT]
  - graph a variety of straight lines using ICT, including parallel lines, and identify similarities and differences (Communicating, Reasoning) [N, ICT, CCT]

Background information

The Cartesian plane is named after Descartes who was one of the first mathematicians to develop analytical geometry on the number plane. He shared this honour with Fermat. Descartes and Fermat are recognised as the first modern mathematicians.

The process of forming right-angled triangles to find gradients is applied in a variety of other situations, eg finding angles of elevation or depression, gradients in calculus in Stage 6, and plotting courses using compass bearings.

Mathematics • Stage 5 (5.1 pathway)

Number and Algebra
Non-linear Relationships

Outcomes
A student:

- uses appropriate terminology, diagrams and symbols in mathematical contexts MA5.1-1WM
- provides reasoning to support conclusions which are appropriate to the context MA5.1-3WM
- graphs simple non-linear relationships MA5.1-7NA

Students:

Graph simple non-linear relations with and without the use of digital technologies and solve simple related equations (ACMNA296)

- complete tables of values to graph simple non-linear relationships and compare with graphs drawn using digital technologies, eg $y = x^2$, $y = x^2 + 2$, $y = 2^x$ [N, ICT]

Explore the connection between algebraic and graphical representations of relations such as simple quadratics, circles and exponentials using digital technology as appropriate (ACMNA239)

- use digital technologies to graph simple quadratics, exponentials and circles [N, ICT],
  eg $y = x^2$, $y = -x^2$, $y = x^2 + 1$, $y = x^2 - 1$
  $y = 2^x$, $y = 3^x$, $y = 4^x$
  $x^2 + y^2 = 1$, $x^2 + y^2 = 4$
  - describe and compare a variety of simple non-linear relationships (Communicating, Reasoning) [N, CCT]
  - connect the shape of a non-linear graph with the distinguishing features of its equation (Communicating, Reasoning) [N, CCT]

Mathematics • Stage 5 (5.1 pathway)

Measurement and Geometry
Area & Surface Area

Outcome
A student:
- uses appropriate terminology, diagrams and symbols in mathematical contexts MA5.1-1WM
- selects and uses appropriate strategies to solve problems MA5.1-2WM
- calculates the area of composite shapes, and the surface area of rectangular and triangular prisms MA5.1-8MG

Related Life Skills outcomes: *MALS-1WM, MALS-2WM, MALS-26MG*

Students:

Calculate the areas of composite shapes (ACMMG216)

- calculate the area of composite figures by dissection into triangles, special quadrilaterals, quadrants, semicircles and sectors [N]
  - ‣ identify different possible dissections for a given composite figure and select an appropriate dissection to facilitate calculation of the area (Problem Solving) [CCT]
- solve practical problems involving area of quadrilaterals and composite shapes [N]
  - ‣ apply properties of geometrical shapes to assist in finding areas, eg symmetry (Problem Solving, Reasoning) [N, CCT]
  - ‣ calculate the area of an annulus (Problem Solving) [N]

Solve problems involving the surface area and volume of right prisms (ACMMG218)

- identify the surface area and edge lengths of rectangular and triangular prisms [L, CCT]
- visualise and name a common solid, given its net [L, CCT]
  - ‣ recognise whether a diagram is a net of a right prism (Reasoning) [CCT]
- visualise and sketch the nets of right prisms [CCT]
- find the surface area of rectangular and triangular prisms, given its net [N]
- calculate the surface area of rectangular and triangular prisms [N]
  - ‣ apply Pythagoras' theorem to assist with finding the surface area of triangular prisms (Problem Solving) [N]
- solve problems involving the surface area of rectangular and triangular prisms (Problem Solving) [N]

Background information

It is important that students can visualise rectangular and triangular prisms in different orientations before they find the surface area. Properties of solids are treated in Stage 3. They should be able to sketch other views of the object.

Mathematics • Stage 5 (5.1 pathway)

Measurement and Geometry
Area & Surface Area

Language

Students could be encouraged to practise expressing the area of a trapezium formula in different ways to improve their understanding and retention of the formula, eg ‘the area of a trapezium is half the perpendicular height times the sum of the parallel sides’, or ‘the area of a trapezium equals half the product of the height and the sum of the parallel sides’.

When calculating the surface area of solids, many students may benefit from writing words to describe each of the faces as they record their calculations. Using words such as top, front, sides and bottom should also assist students to ensure that they include all the faces required.

The abbreviation m^2 is read ‘square metre(s)’ and not ‘metre(s) squared’ or ‘metre(s) square’.
The abbreviation cm^2 is read ‘square centimetre(s)’ and not ‘centimetre(s) squared’ or ‘centimetre(s) square’.

Mathematics • Stage 5 (5.1 pathway)

Measurement and Geometry
Numbers of Any Magnitude

Outcome
A student:
- uses appropriate terminology, diagrams and symbols in mathematical contexts MA5.1-1WM
- selects and uses appropriate strategies to solve problems MA5.1-2WM
- provides reasoning to support conclusions which are appropriate to the context MA5.1-3WM
- interprets very small and very large units of measurement, uses scientific notation, and rounds to significant figures MA5.1-9MG

Students:

Investigate very small and very large time scales and intervals (ACMMG219)

- use the language of estimation appropriately, including:
  - rounding
  - approximate
  - level of accuracy [L, N]
- identify significant figures [L, N]
- round numbers to a specified number of significant figures
- determine the effect of truncating or rounding during calculations on the accuracy of the results [N, CCT]
- interpret the meaning of common prefixes such as ‘milli’, ‘centi’, ‘kilo’ [L]
- interpret the meaning of prefixes for very large and very small units of measurement such as ‘nano’, ‘micro’, ‘mega’, ‘giga’, ‘tera’ [L]
- record measurements of digital information using correct abbreviations, eg kilobytes [kB]
  - ‣ investigate and recognise that some digital devices may use different notations to record measures of digital information, eg 40 kB may appear as 40K or 40k or 40 KB (Communicating) [N] [ICT]
- convert between units of measurement of digital information, eg gigabytes to terabytes, megabytes to kilobytes [N] [ICT]
- use appropriate units of time to measure very small or very large time intervals [L] [N] [CCT]
- describe the limits of accuracy of measuring instruments (±0.5 unit of measurement) [L, N]
  - ‣ explain why measurements are never exact (Communicating, Reasoning) [N, CCT]
  - ‣ appreciate the importance of the number of significant figures in a given measurement (Reasoning) [N, CCT]
  - ‣ choose appropriate units of measurement based on the required degree of accuracy (Communicating, Reasoning) [N, CCT]
  - ‣ consider the degree of accuracy needed when making measurements in practical situations or as a result of calculations (Problem Solving, Reasoning) [N, CCT]

Express numbers in scientific notation (ACMNA210)

- recognise the need for a notation to express very large or very small numbers [L, CCT]
- express numbers in scientific notation
- enter and read scientific notation on a calculator

Mathematics • Stage 5 (5.1 pathway)

Measurement and Geometry
Numbers of Any Magnitude

- explain the difference between numerical expressions such as 2×10^4 and 2^4, particularly with reference to calculator displays (Communicating, Reasoning) [N, CCT]

- use index laws to make order of magnitude checks for numbers in scientific notation, eg $\left(3.12\times10^{4}\right)\times\left(4.2\times10^{6}\right)\approx12\times10^{10}\approx1.2\times10^{11}$ [N]
- convert numbers expressed in scientific notation to decimal form
- order numbers expressed in scientific notation [N]
- solve problems involving scientific notation [N]
  - communicate and interpret technical information using scientific notation (Communicating) [N]

Language

The metric prefixes milli-, centi-, deci- for units smaller than the base SI unit derive from the Latin words: ‘mille’ meaning thousand, ‘centum’ meaning hundred and ‘decimus’ meaning tenth. The metric prefixes kilo-, hecto-, deca- for units larger than the base SI unit derive from the Greek words: *khilioi* meaning thousand, *hekaton* meaning hundred, and *deka* meaning ten.

Mathematics • Stage 5 (5.1 pathway)

Measurement and Geometry

Right-Angled Triangles (Trigonometry)

Outcome

A student:

• uses appropriate terminology, diagrams and symbols in mathematical contexts	MA5.1-1WM
• selects and uses appropriate strategies to solve problems	MA5.1-2WM
• provides reasoning to support conclusions which are appropriate to the context	MA5.1-3WM
• applies trigonometry, given diagrams, to solve problems, including problems involving angles of elevation and depression	MA5.1-10MG

Students:

Use similarity to investigate the constancy of the sine, cosine and tangent ratios for a given angle in right-angled triangles (ACMMG223)

- identify the hypotenuse, adjacent and opposite sides with respect to a given angle in a right-angled triangle in any orientation [L]
  - label sides of right-angled triangles in different orientations in relation to a given angle (Communicating) [L, CCT]
- label the side lengths of a right-angled triangle in relation to a given angle, eg the side *c* is opposite angle *C* [L, CCT]
- define the sine, cosine and tangent ratios for angles in right-angled triangles [L, N]
- recognise the connection between similar triangles and the trigonometric ratios for the angles within the triangles [CCT]
- use trigonometric notation, eg sin*C* [L]
- use a calculator to find approximations of the trigonometric ratios of a given angle measured in degrees [N]
- use a calculator to find an angle correct to the nearest degree, given one of the trigonometric ratios of the angle [N]

Apply trigonometry to solve right-angled triangle problems (ACMMG224)

- select and use appropriate trigonometric ratios in right-angled triangles to find unknown sides, including the hypotenuse [N]
- select and use appropriate trigonometric ratios in right-angled triangles to find unknown angles correct to the nearest degree [N]

Solve right-angled triangle problems including those involving direction and angles of elevation and depression (ACMMG245)

- identify angles of elevation and depression [L]
  - interpret diagrams in questions involving angles of elevation and depression (Reasoning) [CCT]
- solve problems involving angles of elevation and depression when given a diagram [N]
  - solve problems in practical situations involving right-angled triangles, eg finding the pitch of a roof (Problem Solving) [N]

Mathematics • Stage 5 (5.1 pathway)

Measurement and Geometry
Right-Angled Triangles (Trigonometry)

Background information

The definitions of the trigonometric ratios rely on the angle test for similarity, and trigonometry is, in effect, automated calculations with similarity ratios. The content is thus strongly linked with ratio and with scale drawing.

The fact that the other angles and sides of a right-angled triangle are completely determined by giving two other measurements is justified by the four standard congruence tests.

Trigonometry is introduced through similar triangles with students calculating the ratio of two sides and realising that this remains constant for a given angle.

Trigonometry has practical and analytical applications in surveying, navigation, meteorology, architecture, engineering and electronics. It is important to emphasise real-life applications of trigonometry, eg building construction and surveying.

Language

Emphasis should be placed on correct pronunciation of sin as ‘sine’.

Initially students should write the ratio of sides of each of the trigonometric ratios in words, eg $\tan\theta = \dfrac{\text{side opposite angle } \theta}{\text{side adjacent to angle } \theta}$. Abbreviations can be used once students are more familiar with the trigonometric ratios.

When expressing fractions in English, the numerator is said first, followed by the denominator. However, in many Asian languages (eg Chinese, Japanese) the opposite is the case: the denominator is said before the numerator. This may lead to such students mistakenly using the reciprocal of the intended trigonometric ratio and hence students from such language backgrounds should be encouraged to think in English when they are speaking about/expressing fractions.

Students should be explicitly taught the meaning of the phrases ‘angle of elevation’ and ‘angle of depression’. While the meaning of ‘angle of elevation’ may be obvious to many students, the meaning of ‘angle of depression’ as the angle through which a person moves (depresses) their eyes from the horizontal line of sight to look downwards at the required point, may not be as obvious to some students.

Teachers should explicitly demonstrate to students how to deconstruct the large descriptive noun groups frequently associated with descriptions of angles of elevation and depression in word problems, eg ‘The angle of depression of a ship 200 metres out to sea from the top of a cliff is $25°$.’

Students may find some of the terminology/vocabulary encountered in word problems involving trigonometry difficult to interpret, eg ‘base/foot of the mountain’, ‘directly overhead’, ‘pitch of a roof’, or ‘inclination of a ladder’. Teachers should provide students with a variety of word problems and explain such terms explicitly.

The word trigonometry is derived from two Greek words meaning ‘triangle’ and ‘measurement’.

The origin of the word ‘cosine’ is from the Latin *complementi sinus*, meaning ‘complement of sine’, so that $\cos 40° = \sin 50°$.

Mathematics • Stage 5 (5.1 pathway)

Measurement and Geometry
Properties of Geometrical Figures

Outcome
A student:

- uses appropriate terminology, diagrams and symbols in mathematical contexts MA5.1-1WM
- selects and uses appropriate strategies to solve problems MA5.1-2WM
- provides reasoning to support conclusions which are appropriate to the context MA5.1-3WM
- describes and applies the properties of similar figures and scale drawings MA5.1-11MG

Related Life Skills outcomes: *MALS-1WM, MALS-2WM, MALS-3WM, MALS-29MG, MALS-30MG, MALS-31MG*

Students:

Use the enlargement transformation to explain similarity and develop the conditions for triangles to be similar (ACMMG220)

- describe two figures as similar if an enlargement of one is congruent to the other [L]
  - recognise that if two figures are similar they have the same shape but not necessarily the same size (Reasoning)
  - find examples of similar figures embedded in designs from many cultures and historical periods (Reasoning) [N, IU]
  - explain why any two equilateral triangles, or any two squares, are similar, and explain when they are congruent (Communicating, Reasoning) [CCT]
  - investigate whether any two rectangles, or any two isosceles triangles, are similar (Problem Solving) [CCT]
- match the sides and angles of similar figures [N, CCT]
- name the vertices in matching order when using the symbol ||| in a similarity statement [L]
- use the enlargement transformation and measurement to determine that the size of matching angles and the ratio of matching sides are preserved in similar figures [N, CCT]
  - use dynamic geometry software to investigate the properties of similar figures (Problem Solving) [ICT, CCT]

Solve problems using ratio and scale factors in similar figures (ACMMG221)

- choose an appropriate scale in order to enlarge or reduce a diagram [N]
  - enlarge diagrams such as cartoons and pictures (Communicating, Problem Solving) [N]
- construct scale drawings [N]
  - investigate different methods of producing scale drawings, including ICT (Problem Solving) [ICT, CCT]
- interpret and use scales in photographs, plans and drawings found in the media and/or other subjects [N, CCT]
- determine the scale factor for pairs of similar polygons and circles [N]
- calculate dimensions of similar figures using the scale factor [N]
  - apply similarity to finding lengths in the environment where it is impractical to measure directly, eg heights of trees, buildings (Problem Solving) [N, CCT]

Mathematics • Stage 5 (5.1 pathway)

Measurement and Geometry
Properties of Geometrical Figures

- apply the scale factor to find unknown sides in similar triangles [N]
- calculate unknown sides in a pair of similar triangles using a proportion statement [N]

Background information

The definitions of the trigonometric ratios depend upon the similarity of triangles, eg any two right-angled triangles in which another angle is 30° must be similar.

Mathematics • Stage 5 (5.1 pathway)

Statistics and Probability
Single Variable Data Analysis

Outcome
A student:
- uses appropriate terminology, diagrams and symbols in mathematical contexts MA5.1-1WM
- selects and uses appropriate strategies to solve problems MA5.1-2WM
- provides reasoning to support conclusions which are appropriate to the context MA5.1-3WM
- uses statistical displays to compare sets of data, and evaluates statistical claims made in the media MA5.1-12SP

Related Life Skills outcomes: *MALS-1WM, MALS-2WM, MALS-3WM, MALS-32SP, MALS-33SP, MALS-34SP*

Students:

Identify everyday questions and issues involving at least one numerical and at least one categorical variable, and collect data directly from secondary sources (ACMSP228)

- identify and investigate relevant issues involving at least one numerical and at least one categorical variable using information gained from secondary sources, eg the number of hours in a working week for different professions in Australia; the annual rainfall in various parts Australia compared with other countries in the Asia–Pacific region [N, CCT, A]

Construct back-to-back stem-and-leaf plots and histograms and describe data, using terms including 'skewed', 'symmetric' and 'bi modal' (ACMSP282)

- use the terms 'positively skewed', 'negatively skewed', 'symmetric' or 'bi-modal' to describe the shape of distributions of data [L]
  - describe the shape of data displayed in stem-and-leaf plots, dot plots and histograms (Communicating) [L]
  - suggest possible reasons why data distributions may be symmetric, skewed or bi-modal (Reasoning) [CCT]
- construct back-to-back stem-and-leaf plots to display and compare two like sets of numerical data, eg points scored by two sports teams in each game of the season [N, CCT]
  - describe differences in the shapes of the distributions of two sets of like data (Communicating) [L]

Compare data displays using mean, median and range to describe and interpret numerical data sets in terms of location (centre) and spread (ACMSP283)

- interpret two sets of numerical data displayed in back-to-back stem-and-leaf plots, side-by-side dot plots and histograms [N, CCT]
- calculate and compare means, medians and ranges of two sets of numerical data displayed in back-to-back stem-and-leaf plots, side-by-side dot plots and histograms [N, CCT]
  - make comparisons between two like sets of data by referring to the mean, median and/or range, eg 'Team A has a smaller range than Team B, suggesting that Team A is more consistent from week to week than Team B' (Communicating, Reasoning) [N, CCT]

Evaluate statistical reports in the media and other places by linking claims to displays, statistics and representative data (ACMSP253)

- interpret media reports and advertising that quote various statistics, eg media ratings, house prices, sports results, environmental data [L, N, SE]

Mathematics • Stage 5 (5.1 pathway)

Statistics and Probability
Single Variable Data Analysis

- analyse graphical displays to recognise features that may have been manipulated to cause a misleading interpretation and/or support a particular point of view [N, CCT]
  - explain and evaluate the effect of misleading features on graphical displays (Communicating, Reasoning) [N, CCT]
- critically review claims linked to data displays in the media and elsewhere [N, CCT]
  - suggest reasons why data in a display may be misrepresented in the accompanying text (Communicating, Reasoning) [N, CCT]
- consider, informally, the reliability of conclusions from statistical investigations, considering issues such as factors that may have masked the results, the accuracy of measurements taken, and whether the results can be generalised to other situations [N, CCT]

Background information

At this Stage, students are only required to recognise the general shape and lack of symmetry in skewed distributions. No specific analysis of the relative positions of mean, mode and median is required.

Mathematics • Stage 5 (5.1 pathway)

Statistics and Probability
Probability

Outcome
A student:
- uses appropriate terminology, diagrams and symbols in mathematical contexts MA5.1-1WM
- selects and uses appropriate strategies to solve problems MA5.1-2WM
- provides reasoning to support conclusions which are appropriate to the context MA5.1-3WM
- calculates relative frequencies to estimate probabilities of simple and compound events MA5.1-13SP

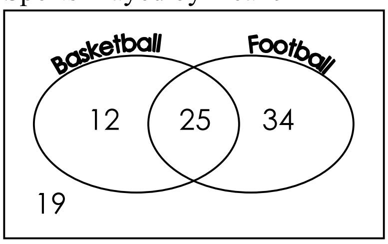
Related Life Skills outcomes: *MALS-1WM, MALS-2WM, MALS-3WM, MALS-35SP, MALS-36SP*

Students:

Calculate relative frequencies from given or collected data to estimate probabilities of events involving 'and' or 'or' (ACMSP226)

- model and repeat a chance experiment a number of times to determine the relative frequencies of outcomes, eg using random number generators such as dice, coins, spinners or digital simulators [L]
    - recognise randomness in chance situations (Reasoning) [N, CCT]
    - recognise that probability estimates become more stable as the number of trials increases (Reasoning) [N, CCT]
- identify theoretical probabilities as being the likelihood of outcomes occurring under ideal circumstances [L]
    - explain the relationship between the relative frequency of an event and its theoretical probability (Communicating, Reasoning) [N, CCT]
- predict the probability of an event from experimental data using relative frequencies [N]
    - apply relative frequency to predict future experimental outcomes (Reasoning, Problem Solving) [N, CCT]
    - design a device to produce a specified relative frequency, eg a four-coloured circular spinner (Problem Solving) [N, CCT]
- calculate probabilities of events, including events involving 'and', 'or' and 'not', from data contained in Venn diagrams of two or three attributes, eg the Venn diagram represents the sports played by Year 9

Sports Played by Year 9



What is the probability that a randomly chosen student plays basketball or football but not both? (Problem Solving) [L, N, CCT]

Mathematics • Stage 5 (5.1 pathway)

Statistics and Probability
Probability

- calculate probabilities of events, including events involving ‘and’, ‘or’ and ‘not’, from data contained in two-way tables, eg the table below represents data collected on Year 10 students comparing gender with handedness

	Left-handed	Right-handed	
Female	7	46	53
Male	5	63	68
	12	109	121

  What is the probability that a randomly chosen student is both female and right-handed? (Problem Solving) [L, N, CCT]

Background information

ICT could be used for simulation experiments to demonstrate that the relative frequency gets closer and closer to the theoretical probability as the number of trials increases.

Students may not appreciate the significance of a simulation, eg they may not transfer results from a digital simulator for tossing a die to the situation of actually tossing a die a number of times.

Mathematics • Stage 5 (5.2 pathway)

Number and Algebra

Financial Mathematics ◊

Outcomes

A student:

- selects appropriate notations and conventions to communicate mathematical ideas and solutions MA5.2-1WM
- interprets mathematical or real-life situations, systematically applying appropriate strategies to solve problems MA5.2-2WM
- solves financial problems involving compound interest MA5.2-4NA

Related Life Skills outcomes: *MALS-1WM, MALS-2WM, MALS-11NA, MALS-12NA, MALS-13NA, MALS-14NA*

Students:

Connect the compound interest formula to repeated applications of simple interest using appropriate digital technologies (ACMNA229)

- establish and use the formula for compound interest,
  $A = P(1+R)^n$ where A is the total amount, P is the principal, R is the interest rate per compounding period as a decimal and n is the number of compounding periods
  - calculate and compare investments for different compounding periods, eg calculate and compare the value of an investment of \$3000 at an interest rate of 6% pa after 5 years when interest is compounded annually as opposed to interest being compounded monthly (Communicating) [N, CCT, PSC]
- solve problems involving compound interest [N]
  - calculate the principal needed to obtain a particular total amount (Problem Solving) [N]
  - use a 'guess and refine' strategy to determine the number of time periods required to obtain a particular total amount (Problem Solving) [N, CCT]
  - compare the cost of loans using flat and reducible interest for a small number of repayment periods (Communicating) [N, CCT, PSC]
- use the compound interest formula to calculate depreciation [N, PSC]

Background information

Internet sites may be used to find commercial rates for home loans and 'home-loan calculators'.

Mathematics • Stage 5 (5.2 pathway)

Number and Algebra
Proportion

Outcomes
A student:
- selects appropriate notations and conventions to communicate mathematical ideas and solutions MA5.2-1WM
- interprets mathematical or real-life situations, systematically applying appropriate strategies to solve problems MA5.2-2WM
- recognises direct and indirect proportion, and solves problems involving direct proportion MA5.2-5NA

Students:

Solve problems involving direct proportion. Explore the relationship between graphs and equations corresponding to simple rate problems (ACMNA208)

- convert between units for rates , eg kilometres per hour to metres per second [N]
- identify and describe everyday examples of direct proportion, eg as the number of hours worked increases, earnings also increase [N]
- identify and describe everyday examples of inverse (indirect) proportion, eg as speed increases, the time taken to travel a particular distance decreases [N]
- recognise direct and inverse proportion from graphs [N]
  - distinguish between positive and negative gradients from a graph (Reasoning) [N, CCT]
- interpret and use conversion graphs to convert from one unit to another, eg conversions between different currencies or metric and imperial measures [N]
- use the equation $y = kx$ to model direct linear proportion where k is the constant of proportionality [N]
  - given the constant of proportionality, establish an equation and use it to find an unknown quantity (Communicating, Problem Solving) [N]
  - calculate the constant of proportionality, given appropriate information, and use this to find unknown quantities (Problem Solving) [N]
- use graphing software or a table of values to graph equations of linear direct proportion [ICT]

Mathematics • Stage 5 (5.2 pathway)

Number and Algebra
Algebraic Techniques

Outcomes
A student:
- selects appropriate notations and conventions to communicate mathematical ideas and solutions MA5.2-1WM
- constructs arguments to prove and justify results MA5.2-3WM
- simplifies algebraic fractions, and expands and factorises quadratic expressions MA5.2-6NA

Students:

Apply the four operations to simple algebraic fractions with numerical denominators (ACMNA232)

- simplify expressions that involve algebraic fractions with numerical denominators, eg $\frac{a}{2}+\frac{a}{3}$, $\frac{2x}{5}-\frac{x}{3}$, $\frac{3x}{4}\times\frac{2x}{9}$, $\frac{3x}{4}\div\frac{9x}{2}$
  - connect the processes for simplifying algebraic fractions with the processes for numerical fractions (Communicating) [CCT]

Apply the four operations to algebraic fractions

- simplify algebraic fractions, including those with indices, eg $\frac{10a^4}{5a^2}$, $\frac{9a^2b}{3ab}$, $\frac{3ab}{9a^2b}$
  - explain the difference between expressions such as $\frac{3a}{9}$ and $\frac{9}{3a}$ (Communicating) [L, CCT]
- simplify expressions that involve algebraic fractions, including those with algebraic denominators and/or indices, eg $\frac{2ab}{3}\times\frac{6}{2b}$, $\frac{3x^2}{8y^5}\div\frac{15x^3}{4y}$, $\frac{a^2b^4}{6}\times\frac{9}{a^2b^2}$, $\frac{3}{x}-\frac{1}{2x}$

Apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate (ACMNA213)

- expand algebraic expressions, including those involving negatives and indices, eg $-3x^2\left(5x^2+2x^4y\right)$
- expand algebraic expressions by removing grouping symbols and collecting like terms where applicable, eg $2y\left(y-5\right)+4\left(y-5\right)$, $4x\left(3x+2\right)-\left(x-1\right)$

Factorise algebraic expressions by taking out a common algebraic factor (ACMNA230)

- factorise algebraic expressions by determining common factors, eg $3x^2-6x$, $14ab+12a^2$, $21xy-3x+9x^2$, $15p^2q^3-12pq^4$
  - recognise that expressions such as $24x^2y+16xy^2=4xy\left(6x+4y\right)$ are partially factorised and that further factorisation is necessary (Reasoning) [CCT]

Mathematics • Stage 5 (5.2 pathway)

Number and Algebra
Algebraic Techniques

Expand binomial products and factorise monic quadratic expressions using a variety of strategies (ACMNA233)

- expand binomial products by finding the area of rectangles [N],

	x	8
x	x^2	$8x$
3	$3x$	24

  hence, $\left(x+8\right)\left(x+3\right) = x^2 + 8x + 3x + 24$
  $= x^2 + 11x + 24$

- use algebraic methods to expand binomial products, eg $\left(x+2\right)\left(x-3\right)$, $\left(4a-1\right)\left(3a+2\right)$

- factorise monic quadratic expressions, eg x^2+5x+6, x^2+2x-8
  - connect binomial products with the commutative property of arithmetic such that $\left(a+b\right)\left(c+d\right)=\left(c+d\right)\left(a+b\right)$ (Communicating) [CCT]
  - explain why a particular algebraic expansion or factorisation is incorrect, eg 'Why is the factorisation $x^2-6x-8=\left(x-4\right)\left(x-2\right)$ incorrect?' (Communicating, Reasoning) [L, CCT]

Mathematics • Stage 5 (5.2 pathway)

Number and Algebra
Indices

Outcomes
A student:
- selects appropriate notations and conventions to communicate mathematical ideas and solutions MA5.2-1WM
- constructs arguments to prove and justify results MA5.2-3WM
- applies index laws to operate with algebraic expressions involving integer indices MA5.2-7NA

Students:

Apply index laws to algebraic expressions with integer indices

- use index notation and the index laws to establish that $a^{-1} = \frac{1}{a}$, $a^{-2} = \frac{1}{a^2}$, $a^{-3} = \frac{1}{a^3}$, $\ldots$ [CCT]
  - ‣ explain the difference between similar pairs of algebraic expressions, eg 'Are x^{-2} and $-2x$ equivalent expressions?' (Communicating) [CCT]
- translate expressions with negative indices to expressions with positive indices and vice versa
- apply the index laws to simplify algebraic expressions involving negative indices, eg $4b^{-5} \times 8b^{-3}$, $9x^{-4} \div 3x^3$
  - ‣ state whether particular equivalences are true or false and give reasons, eg explain why each of the following are true or false: $5x^0 = 1$, $9x^5 \div 3x^5 = 3x$, $a^5 \div a^7 = a^2$, $2c^{-4} = \frac{1}{2c^4}$ (Communicating, Reasoning) [CCT]
  - ‣ determine and justify whether a simplified expression is correct by substituting numbers for pronumerals (Communicating, Reasoning) [CCT]
  - ‣ evaluate a numerical fraction raised to the power of −1, leading to $\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$ (Communicating, Reasoning) [CCT]

Mathematics • Stage 5 (5.2 pathway)

Number and Algebra
Equations

Outcomes
A student:
- selects appropriate notations and conventions to communicate mathematical ideas and solutions MA5.2-1WM
- interprets mathematical or real-life situations, systematically applying appropriate strategies to solve problems MA5.2-2WM
- constructs arguments to prove and justify results MA5.2-3WM
- solves linear and simple quadratic equations, linear inequalities and linear simultaneous equations, using analytical and graphical techniques MA5.2-8NA

Related Life Skills outcomes: *MALS-1WM, MALS-2WM, MALS-3WM, MALS-16NA*

Students:

Sketch linear graphs using the coordinates of two points and solve linear equations (ACMNA215)

- solve linear equations, including equations that involve grouping symbols, eg $3(a+2)+2(a-5)=10$, $3(2m-5)=2m+5$

Solve linear equations involving simple algebraic fractions (ACMNA240)

- solve linear equations involving one or more fractions, eg $\frac{x-2}{3}+5=10$, $\frac{2x+5}{3}=10$, $\frac{2x}{3}+5=10$, $\frac{x}{3}+\frac{x}{2}=5$, $\frac{2x+5}{3}=\frac{x-1}{4}$
  - compare and contrast different methods of solving linear equations and justify a choice for a particular case (Communicating, Reasoning) [N, CCT]

Solve simple quadratic equations using a range of strategies (ACMNA241)

- solve simple quadratic equations of the form $ax^2=c$, leaving answers in exact form or as decimal approximations
  - explain why quadratic equations could be expected to have two solutions (Communicating, Reasoning) [CCT]
  - recognise and explain that $x^2=c$ does not a have solution if c is a negative number (Communicating) [CCT]
- solve quadratic equations of the form $ax^2+bx+c=0$, limited to $a=1$, using factors
  - connect algebra with arithmetic, to explain why if $p\times q=0$, then either $p=0$ or $q=0$ (Communicating, Reasoning) [N, CCT]
  - check the solution(s) of quadratic equations by substitution (Reasoning) [CCT]

Substitute values into formulas to determine an unknown (ACMNA234)

- solve equations arising from substitution into formulas, eg given $P=2l+2b$ and $P=20, l=6$, solve for b
  - substitute into formulas from other strands of the syllabus or in other subjects to solve problems and interpret solutions, eg $A=\frac{1}{2}xy$, $v=u+at$, $C=\frac{5}{9}(F-32)$, $V=\pi r^2h$ (Problem Solving) [N]

Mathematics • Stage 5 (5.2 pathway)

Number and Algebra
Equations

Solve problems involving linear equations, including those derived from formulas (ACMNA235)

- translate word problems into equations, solve the equations and interpret the solutions [N]
  - state clearly the meaning of introduced unknowns as 'the number of …' (Communicating)
  - solve word problems involving familiar formulas, eg 'The area of a triangle is 30 square centimetres and base length 12 centimetres, find the perpendicular height of the triangle.' (Problem Solving) [L, N]
  - explain why the solution to a linear equation generated from a word problem may not be a solution to the given problem (Communicating, Reasoning) [CCT]

Solve linear inequalities and graph their solutions on a number line (ACMNA236)

- represent simple inequalities on the number line, eg $x \le 4$, $m > -3$ [N]
- recognise that an inequality has an infinite number of solutions [N]
- solve linear inequalities, including reversing the direction of the inequality when multiplying or dividing by a negative number, and graph the solutions,
  eg $3x-1<9$, $2(a+4)\ge 24$, $\frac{t+4}{5} > -3$, $1-4y \le 6$ [N]
  - use a numerical example to justify the need to reverse the direction of the inequality when multiplying or dividing by a negative number (Reasoning) [CCT]
  - verify the direction of the inequality sign by substituting a value within the solution range (Reasoning) [CCT]

Solve linear simultaneous equations, using algebraic and graphical techniques including using digital technology (ACMNA237)

- solve linear simultaneous equations by finding the point of intersection of their graphs with and without ICT [ICT, N]
- solve linear simultaneous equations using an appropriate algebraic method,
  eg solve $\begin{cases} 3a+b=17 \\ 2a-b=8 \end{cases}$
- generate and solve linear simultaneous equations from word problems and interpret the results [N, CCT]

Background information

Graphics calculators and graphing software enable students to graph two linear equations and display the coordinates of the point of intersection.

Mathematics • Stage 5 (5.2 pathway)

Number and Algebra
Linear Relationships

Outcomes

A student:

- selects appropriate notations and conventions to communicate mathematical ideas and solutions — MA5.2-1WM
- constructs arguments to prove and justify results — MA5.2-3WM
- uses the gradient-intercept form to interpret and graph linear relationships — MA5.2-9NA

Students:

Describe, interpret and sketch linear relationships using gradient-intercept form

- graph straight lines of the form $y = mx + b$ (gradient/intercept form) [N]
- recognise equations of the form $y = mx + b$ as representing straight lines and interpret the x-coefficient (m) as the gradient and the constant (b) as the y-intercept [L, N]
- rearrange an equation in general form $ax + by + c = 0$ (general form) to gradient/intercept form to determine the gradient and y-intercept
- find the equation of a straight line using $y = mx + b$ given the gradient and y-intercept
- graph equations of the form $y = mx + b$, with and without ICT, and by using the y-intercept and gradient [N, ICT]
  - use graphing software to graph a variety of equations of straight lines, and describe the similarities and differences between them,
    eg $y = -3x$, $y = -3x + 2$, $y = -3x - 2$
    $y = \frac{1}{2}x$, $y = -2x$, $y = 3x$
    $x = 2$, $y = 2$
    (Communicating) [N, ICT, CCT]
  - explain the effect of changing the gradient or y-intercept on the graph of a line (Communicating) [CCT]
- find the gradient and the y-intercept of a straight line from the graph and use these to determine the equation of the line [N]
  - match equations of straight lines to graphs of straight lines and justify choices (Communicating, Reasoning) [N, CCT]

Solve problems involving parallel and perpendicular lines (ACMNA238)

- determine that lines are perpendicular if the product of their gradients is -1
  - graph a variety of lines using ICT, including perpendicular lines, and compare their gradients to establish the condition for lines to be perpendicular (Communicating) [N, ICT, CCT]
  - recognise that when lines are perpendicular, the gradient of one line is the negative reciprocal of the other (Reasoning) [CCT]
- find the equation of a straight line parallel or perpendicular to another given line using $y = mx + b$ [N]

Mathematics • Stage 5 (5.2 pathway)

Number and Algebra

Non-linear Relationships ◊

Outcomes

A student:

- selects appropriate notations and conventions to communicate mathematical ideas and solutions MA5.2-1WM
- constructs arguments to prove and justify results MA5.2-3WM
- connects algebraic and graphical representations of simple non-linear relationships MA5.2-10NA

Students:

Graph simple non-linear relations with and without the use of digital technologies and solve simple related equations (ACMNA296)

- graph parabolic relationships of the form $y = ax^2$, $y = ax^2 + c$ [N]
  - ‣ identify parabolic shapes in the environment (Reasoning) [N]
  - ‣ describe the effect on the graph of $y = x^2$ of multiplying by or adding different constants, including negatives (Communicating) [N, CCT]
  - ‣ determine the equation of a parabola given a graph with the main features clearly indicated (Reasoning) [L, N, CCT]
- determine the x-coordinate of a point on a parabola given the y-coordinate
- sketch, compare and describe the key features of simple exponential curves, with and without ICT, eg $y = 2^x$, $y = -2^x$, $y = 2^{-x}$, $y = -2^{-x}$, $y = 2^x + 1$, $y = 2^x - 1$ [L, N, CCT]
  - ‣ describe exponentials in terms of what happens to the y-values as x becomes very large or very small and what occurs at $x = 0$ (Communicating, Reasoning) [N, CCT]
- recognise and describe equations that represent circles with centre the origin and radius r [L, N]
  - ‣ use Pythagoras' theorem to establish the equation of a circle, centre the origin, radius r and graph equations of the form $x^2 + y^2 = r^2$ (Reasoning) [N]
- sketch circles of the form $x^2 + y^2 = r^2$ where r is the radius [L, N]

Explore the connection between algebraic and graphical representations of relations such as simple quadratics, circles and exponentials using digital technology as appropriate (ACMNA239)

- identify graphs and equations of straight lines, parabolas, circles and exponentials [L, N]
- match graphs of straight lines, parabolas, circles and exponentials to the appropriate equations [N, CCT]
  - ‣ sort and classify different types of graphs, match each graph to an equation, and justify each choice (Communicating, Reasoning) [N, CCT]

Mathematics • Stage 5 (5.2 pathway)

Number and Algebra

Non-linear Relationships ◊

Background information

Graphics calculators and graphing software facilitate the investigation of the shapes of curves and the effect on the equation of multiplying by, or adding, a constant.

This substrand could provide opportunities for modelling. For example, $y = 1.2^x$ for $x \geq 0$, models the growth of a quantity beginning at 1 and increasing 20% for each unit increase in *x*.

Mathematics • Stage 5 (5.2 pathway)

Measurement and Geometry
Area & Surface Area

Outcome
A student:
- selects appropriate notations and conventions to communicate mathematical ideas and solutions — MA5.2-1WM
- interprets mathematical or real-life situations, systematically applying appropriate strategies to solve problems — MA5.2-2WM
- calculates the surface area of right prisms, cylinders and related composite solids — MA5.2-11MG

Related Life Skills outcomes: *MALS-1WM, MALS-2WM, MALS-26MG*

Students:

Calculate the surface area and volume of cylinders and solve related problems (ACMMG217)

- develop and use the formula to find the surface area of closed right cylinders
  Surface area of cylinder $= 2\pi r^2 + 2\pi rh$
  where r is the length of the radius and h is the perpendicular height [N, CCT]
- solve practical problems involving surface area of cylinders, eg find the area of the label of a cylindrical can [N]

Solve problems involving surface area and volume for a range of prisms, cylinders and composite solids (ACMMG242)

- calculate the surface area of composite solids involving right cylinders and prisms [N]
- solve practical problems related to surface area, eg compare the amount of packaging material needed for different objects [N]
  - interpret the given conditions of a problem to determine whether a prism or cylinder is open or closed (Problem Solving) [CCT]

Mathematics • Stage 5 (5.2 pathway)

Measurement and Geometry

Volume

Outcome

A student:

- selects appropriate notations and conventions to communicate mathematical ideas and solutions — MA5.2-1WM
- interprets mathematical or real-life situations, systematically applying appropriate strategies to solve problems — MA5.2-2WM
- applies formulas to calculate the volume of composite solids composed of right prisms and cylinders — MA5.2-12MG

Related Life Skills outcomes: *MALS-1WM, MALS-2WM, MALS-25MG, MALS-27MG, MALS-28MG*

Students:

Solve problems involving the surface area and volume of right prisms (ACMMG218)

- calculate the volume of prisms with cross-sections that are composite figures that may be dissected into triangles and special quadrilaterals [N]
  - ‣ solve practical problems related to volume and/or capacity of composite prisms (Problem Solving) [N]
  - ‣ compare the surface areas of prisms with the same volume (Problem Solving, Reasoning) [N, CCT]
  - ‣ find the volume and/or capacity of various everyday containers such as water tanks or moving cartons (Problem Solving) [N]

Solve problems involving surface area and volume for a range of prisms, cylinders and composite solids (ACMMG242)

- find the volume of solids that have sectors as the uniform cross-section [N]
- find the volume of simple composite solids such as a cylinder on top of a rectangular prism [N]
  - ‣ dissect composite solids into several simpler solids to find volume (Communicating) [N]
- solve practical problems related to volume and/or capacity of prisms, cylinders and composite solids [N]

Mathematics • Stage 5 (5.2 pathway)

Measurement and Geometry

Right-Angled Triangles (Trigonometry) ◊

Outcome

A student:

- selects appropriate notations and conventions to communicate mathematical ideas and solutions MA5.2-1WM
- interprets mathematical or real-life situations, systematically applying appropriate strategies to solve problems MA5.2-2WM
- applies trigonometry to solve problems, including problems involving bearings MA5.2-13MG

Students:

Apply trigonometry to solve right-angled triangle problems (ACMMG224)

- use a calculator to find trigonometric ratios of a given approximation for angles measured in degrees and minutes [N]
- use a calculator to find an approximation for an angle in degrees and minutes, given the trigonometric ratio of the angle [N]
- find unknown sides in right-angled triangles where the given angle is measured in degrees and minutes [N]
- use trigonometric ratios to find unknown angles in degrees and minutes in right-angled triangles [N]

Solve right-angled triangle problems including those involving direction and angles of elevation and depression (ACMMG245)

- use three-figure bearings (eg 035°, 225°) and compass bearings (eg SSW) [L, N]
  - interpret directions given as bearings and represent them in diagrammatic form (Communicating) [N, CCT]
- solve word problems involving bearings or angles of elevation and depression [N]
  - draw diagrams to assist in solving practical problems involving bearings, angles of elevation and depression (Communicating) [CCT]
  - check the reasonableness of answers to trigonometry problems (Problem Solving) [N, CCT]

Background information

Students may need encouragement to set out their solutions carefully and to use the correct mathematical language and suitable diagrams.

When setting out their solutions related to finding unknown lengths and angles, students should be advised to give a simplified exact answer, eg 25sin42° metres or $\sin A = \frac{4}{7}$, then give an approximation correct to a specified or sensible level of accuracy.

Students could have practical experience in using clinometers for finding angles of elevation and depression and in using magnetic compasses for bearings. Students need to recognise the 16 points of a mariner’s compass (eg SSW) for comprehension of compass bearings in everyday life, eg weather reports.

Students studying circle geometry will be able to apply their trigonometry to many problems, making use of the right-angles between a chord and a radius bisecting it, between a tangent and a radius at the point of contact, and in a semicircle.

Mathematics • Stage 5 (5.2 pathway)

Measurement and Geometry

Right-Angled Triangles (Trigonometry) ◊

Language

Students need to be able to interpret a variety of phrases involving bearings, such as:

- ‘the bearing of Melbourne from Sydney is 230°’
- ‘a plane flies to Melbourne on a bearing of 230° from Sydney’
- ‘a plane flies from Sydney to Melbourne on a bearing of 230°’
- ‘a plane takes off from Sydney and flies on a bearing of 230° to Melbourne’.

Students should be taught explicitly how to identify the location from where the bearing is measured and to draw the centre of the compass rose at this location in their diagram. In each of the examples above, the word ‘from’ indicates that the bearing has been measured in Sydney and, consequently, in a diagram, the centre of the compass rose is at Sydney.

To help students understand questions that reference a path involving more than one bearing, they may need to be explicitly shown to look for words such as ‘after this’, ‘then’, ‘changes direction’ that indicate a change of bearing and thus that a new compass rose needs to be drawn at the location of each change in direction.

Mathematics • Stage 5 (5.2 pathway)

Measurement and Geometry
Properties of Geometrical Figures

Outcome
A student:
- selects appropriate notations and conventions to communicate mathematical ideas and solutions MA5.2-1WM
- interprets mathematical or real-life situations, systematically applying appropriate strategies to solve problems MA5.2-2WM
- constructs arguments to prove and justify results MA5.2-3WM
- calculates the angle sum of any polygon and uses minimum conditions to prove triangles are congruent MA5.2-14MG

Students:

Formulate proofs involving congruent triangles and angle properties (ACMMG243)

- write formal proofs of congruence of triangles preserving matching order of vertices [L, CCT]
- apply congruent triangle results to prove properties of isosceles and equilateral triangles [CCT]:
  - if two sides of a triangle are equal, then the angles opposite the equal sides are equal
  - conversely, if two angles of a triangle are equal, then the sides opposite those angles are equal
  - if three sides of a triangle are equal then each interior angle is 60°
- use congruent triangles to prove properties of the special quadrilaterals [CCT], such as:
  - opposite angles of a parallelogram are equal
  - diagonals of a parallelogram bisect each other
  - diagonals of a rectangle are equal in length

Use the enlargement transformation to explain similarity and develop the conditions for triangles to be similar (ACMMG220)

- investigate the minimum conditions needed, and establish the four tests, for two triangles to be similar [L, CCT]
  - if the three sides of one triangle are proportional to the three sides of another triangle, then the two triangles are similar
  - if two sides of one triangle are proportional to two sides of another triangle, and the included angles are equal, then the two triangles are similar
  - if two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar
  - if the hypotenuse and a second side of a right-angled triangle are proportional to the hypotenuse and a second side of another right-angled triangle, then the two triangles are similar
- determine whether two triangles are similar using an appropriate test [CCT]

Apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes (ACMMG244)

- apply geometrical facts, properties and relationships to find unknown sides and angles in diagrams, providing appropriate reasons [N, CCT]
  - ‣ recognise that more than one method of solution is possible (Reasoning) [CCT]
  - ‣ compare different solutions for the same problem to determine the most efficient method (Communicating, Reasoning) [CCT]

Mathematics • Stage 5 (5.2 pathway)

Measurement and Geometry

Properties of Geometrical Figures

- ‣ apply the properties of congruent and similar triangles, justifying the results (Communicating, Reasoning) [CCT]
- define the exterior angle of a convex polygon [L]
- establish that the sum of the exterior angles of any convex polygon is 360° [CCT]
  - ‣ use dynamic geometry software to investigate the constancy of the exterior angle sum of polygons for different polygons (Reasoning) [ICT, CCT]
- apply the result for the interior angle sum of a triangle to find, by dissection, the interior angle sum of polygons with more than three sides [N]
  - ‣ use dynamic geometry software to investigate the angle sum of different polygons (Reasoning) [ICT, CCT]
  - ‣ express in algebraic terms the interior angle sum of a polygon with n sides, eg interior angle sum $= (n-2) \times 180°$ (Communicating) [N, CCT]
- apply angle sum results to find unknown angles in polygons [N, CCT]

Background information

Students are expected to give reasons when proving properties of plane shapes using congruence.

Dynamic geometry software or prepared applets are useful tools for investigating the interior and exterior angle sums of polygons.

The concept of the exterior angle sum of a convex polygon may be interpreted as the amount of turning required during a circuit of the boundary.

Comparing the perimeters of inscribed and circumscribed polygons leads to an approximation for the circumference of a circle. This is the method Archimedes used to develop an approximation for the ratio of the circumference to the diameter, that is, π.

Mathematics • Stage 5 (5.2 pathway)

Statistics and Probability

Single Variable Data Analysis ◊

Outcome

A student:

- selects appropriate notations and conventions to communicate mathematical ideas and solutions MA5.2-1WM
- constructs arguments to prove and justify results MA5.2-3WM
- uses quartiles and box plots to compare sets of data, and evaluates sources of data MA5.2-15SP

Related Life Skills outcomes: *MALS-1WM, MALS-3WM, MALS-32SP, MALS-33SP, MALS-34SP*

Students:

Determine quartiles and interquartile range (ACMSP248)

- determine the upper and lower extremes, median, and upper and lower quartiles for sets of numerical data [L, N]
  - describe the proportion of scores contained between various quartiles, eg 75% of scores lie between the lower quartile and upper extreme (Problem Solving, Reasoning) [N]
- determine the interquartile range for a set of scores [L]
  - recognise that the interquartile range is a measure of spread of the middle 50% of data (Reasoning) [N, CCT]
- compare the relative merits of the range and interquartile range as measures of spread [N, CCT]
  - explain why the range or interquartile range is a better measure of spread for particular data sets, eg when the data contains an outlier (Communicating, Reasoning) [N, CCT]

Construct and interpret box plots and use them to compare data sets (ACMSP249)

- construct a box plot using the median, the upper and lower quartiles and the extreme values (the ‘five-point summary’) [N]
- compare two or more sets of data using parallel box plots drawn on the same scale [N, CCT]
  - describe similarities and differences between two data sets displayed in parallel box plots, eg describe differences in spread using interquartile range, and suggest reasons for such differences (Communicating, Reasoning) [N, CCT]

Compare shapes of box plots to corresponding histograms and dot plots (ACMSP250)

- determine quartiles from data displayed in histograms and dot plots, and use these to draw a box plot to represent the same data set [CCT]
  - compare the relative merits of a box plot with its corresponding histogram or dot plot (Reasoning) [CCT]
- identify skewed and symmetrical data sets displayed in histograms and dot plots, and describe the shape/features of the corresponding box plot for such types of data [N, CCT]

Investigate reports of surveys in digital media and elsewhere for information on how data were obtained to estimate population means and medians (ACMSP227)

- investigate survey data reported in the digital media and elsewhere to critically evaluate the reliability/validity of the source of the data and its usefulness [ICT, CCT, EU]

Mathematics • Stage 5 (5.2 pathway)

Statistics and Probability

Single Variable Data Analysis ◊

- ‣ describe bias that may exist due to how the data was obtained, eg who instigated and/or funded the research, the types of survey questions asked, the sampling method used (Reasoning) [N, CCT]

- make predictions from a sample that may apply to the whole population [N]
  - ‣ consider the size of the sample when making predictions about the population (Reasoning) [N, CCT]

Background information

Graphics calculators and other statistical software will display box plots for entered data, but students should be aware that results may not always be the same since the technologies use varying methods of creating the plots.

Mathematics • Stage 5 (5.2 pathway)

Statistics and Probability

Bivariate Data Analysis

Outcome

A student:

- selects appropriate notations and conventions to communicate mathematical ideas and solutions MA5.2-1WM
- constructs arguments to prove and justify results MA5.2-3WM
- investigates relationships between two statistical variables, including their relationship over time MA5.2-16SP

Related Life Skills outcomes: *MALS-1WM, MALS-3WM, MALS-35SP, MALS-36SP*

Students:

Investigate and describe bivariate numerical data where the independent variable is time (ACMSP252)

- recognise the difference between an independent variable and its dependent variable [L]
- distinguish bivariate data from single variable (univariate) data [L]
  - describe the difference between bivariate data and single variable data using an appropriate example, eg bivariate data compares two variables such as arm span and height, while single variable data examines only one variable such as arm span (Communicating) [CCT]
- investigate an issue of interest, representing the dependent numerical variable against the independent variable, time, in an appropriate graphical form [N]
  - determine and explain why line graphs are the most appropriate method of representing data collected over time (Reasoning) [N, CCT]
  - describe changes in the dependent variable over time, eg describe changes in carbon pollution over time (Communicating) [N, CCT, SE]
  - suggest reasons for changes in the dependent variable over time with reference to relevant world or national events, eg describe the change in population of Australia over time with respect to historical events (Reasoning) [N, CCT, AHC]
- interpret data displays representing two or more dependent numerical variables against time, eg compare daily food intake of different countries over time [N, CCT, IU]

Use scatter plots to investigate and comment on relationships between two numerical variables (ACMSP251)

- investigate an issue of interest involving two numerical variables and construct a scatter plot, with or without the use of ICT, to determine and comment on the relationship between them, eg height versus arm span; reaction time versus hours of sleep [N, ICT, CCT]
- describe, informally, the strength and direction of the relationship between two variables displayed in a scatter plot, eg strong positive relationship, weak negative relationship, no association [N, CCT]
- make predictions from a given scatter diagram or graph [N, CCT]

Mathematics • Stage 5 (5.2 pathway)

Statistics and Probability
Probability

Outcome
A student:
- selects appropriate notations and conventions to communicate mathematical ideas and solutions MA5.2-1WM
- interprets mathematical or real-life situations, systematically applying appropriate strategies to solve problems MA5.2-2WM
- constructs arguments to prove and justify results MA5.2-3WM
- describes and calculates probabilities in multi-step chance experiments MA5.2-17SP

Related Life Skills outcomes: *MALS-1WM, MALS-2WM, MALS-3WM, MALS-35SP, MALS-36SP*

Students:

List all outcomes for two-step chance experiments, both with and without replacement using tree diagrams or arrays. Assign probabilities to outcomes and determine probabilities for events (ACMSP225)

- sample with and without replacement in two-step chance experiments, eg drawing two counters from a bag containing three blue, four red and one white counter [L, N]
  - compare results between an experiment undertaken firstly with replacement and then without (Reasoning) [N, CCT]
- record outcomes of two-step chance experiments with and without replacement using organised lists, tables and tree diagrams [N, CCT]
- calculate probabilities of simple and compound events in two-step chance experiments, both with and without replacement [N]
  - explain the effect of knowing the result of the first step on the probability of events in two-step chance experiments with or without replacement (Communicating, Reasoning) [N, CCT]

Describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Investigate the concept of independence (ACMSP246)

- distinguish informally between dependent and independent events [L]
  - explain the difference between dependent and independent events using appropriate examples (Communicating, Reasoning) [L, CCT]
- recognise that for independent events $P(A \text{ and } B) = P(A) \times P(B)$ [N, CCT]
- sample with and without replacement in three-step chance experiments, eg tossing three coins or drawing three counters from a bag containing three blue, four red and one white counter [N]
- record outcomes of three-step chance experiments with and without replacement using organised lists, tables or tree diagrams [N, CCT]
- calculate probabilities of simple and compound events in three-step chance experiments, both with and without replacement [N]
  - use knowledge of complementary events to assist with calculating probabilities of events in multi-step chance experiments (Problem Solving)
  - evaluate the likelihood of winning a prize in lotteries and other competitions (Problem Solving, Reasoning) [N, CCT]

Mathematics • Stage 5 (5.2 pathway)

Statistics and Probability
Probability

Use the language of 'if ... then', 'given', 'of', 'knowing that' to investigate conditional statements and identify common mistakes in interpreting such language (ACMSP247)

- calculate probabilities of events where a condition is given that restricts the sample space, eg given that a number less than 5 has been rolled on a fair six-sided die, calculate the probability that a 3 was rolled [CCT]
  - describe the effect of a given condition on the sample space, eg in the above example, the sample space is reduced to $\{1, 2, 3, 4\}$ (Communicating, Problem Solving, Reasoning) [CCT]
- critically evaluate conditional statements used in descriptions of chance situations [N, CCT]
  - describe the validity of conditional statements used in descriptions of chance situations with reference to dependent and independent events, eg explain why if you toss a coin and obtain a head, then the chance of obtaining a head on the next toss remains the same (Communicating, Reasoning) [N, CCT]
  - identify and explain common misconceptions related to chance experiments, eg explain why the statement, 'If you obtain a tail in each of four consecutive tosses of a coin, then there is a greater chance of obtaining a head with the next toss' is incorrect (Reasoning) [N, CCT]

Background information

Meteorologists use probability to predict the weather, eg chance of rain. Insurance companies use probability to determine premiums, eg chance of particular age groups having accidents.

The mathematical analysis of probability was prompted by the French gentleman gambler, the Chevalier de Méré. Over the years, the Chevalier had consistently won money betting on at least one six in four rolls of a die. He felt that he should also win betting on at least one double six in 24 rolls of two dice, but in fact regularly lost.

In 1654 he asked his mathematician friend Pascal to explain why. This question led to a famous correspondence between Pascal and the renowned mathematician Fermat. Chevalier's change of fortune is explained by the fact that the chance of at least one six in four rolls of a die is $1-\left(\frac{5}{6}\right)^4 = 51.8\%\ (1\text{ dp})$, while the chance of at least one double six in 24 rolls of two dice is $1-\left(\frac{35}{36}\right)^{24} = 49.1\%\ (1\text{ dp})$.

Language

In a chance experiment, such as rolling a fair six-sided die twice, an event is a collection of outcomes. For instance, an event might be that the result is 'a sum of 7', or 'a sum of 10 or more'.

Mathematics • Stage 5 (5.3 pathway)

Number and Algebra
Proportion

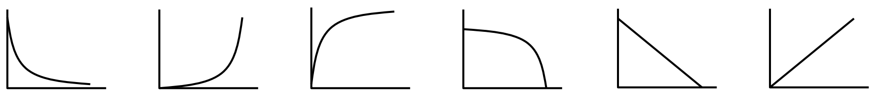
Outcomes
A student:

- uses and interprets formal definitions and generalisations when explaining solutions and/or conjectures — MA5.3-1WM
- generalises mathematical ideas and techniques to analyse and solve problems efficiently — MA5.3-2WM
- uses deductive reasoning in presenting arguments and formal proofs — MA5.3-3WM
- draws, interprets and analyses graphs of physical phenomena — MA5.3-4NA

Students:

Solve problems involving direct proportion. Explore the relationship between graphs and equations corresponding to simple rate problems (ACMNA208)

- interpret distance/time graphs when the speed is variable [N]
  - match a set of distance/time graphs to situations, and explore the likelihood that they are accurate, appropriate, and whether they are possible (Problem Solving, Reasoning) [N]
  - match a set of distance/time graphs to a set of descriptions and give reasons for choices (Communicating, Reasoning) [L, N, CCT]
  - record the distance of a moving object from a fixed point at equal time intervals and draw a graph to represent the situation, eg move along a measuring tape for 30 seconds using a variety of activities that include variable speeds such as running fast, walking slowly, and walking slowly then speeding up (Communicating, Problem Solving) [N]
- analyse the relationship between variables as they change over time, eg draw graphs to represent the relationship between the depth of water in containers of different shapes when they are filled at a constant rate [N, CCT]
- interpret graphs, making sensible statements about rate of increase or decrease, the initial and final points, constant relationships as denoted by straight lines, variable relationships as denoted by curved lines, etc [N, CCT]
  - decide whether a particular graph is a suitable representation of a given physical phenomenon (Communicating) [N, CCT]
- describe qualitatively the rate of change of a graph using terms such as ‘increasing at a decreasing rate’ [N, CCT]



decreasing at a decreasing rate	increasing at an increasing rate	increasing at a decreasing rate	decreasing at an increasing rate	decreasing at a constant rate	increasing at a constant rate

- sketch a graph from a simple description given a variable rate of change [N]

Mathematics • Stage 5 (5.3 pathway)

Number and Algebra

Proportion

Background information

Rate of change is considered as it occurs in practical situations, including population growth and travel. Simple linear models have a constant rate of change. In other situations, the rate of change is variable.

This work is intended to provide experiences for students that will give them an intuitive understanding of rates of change and will assist the development of appropriate vocabulary. No quantitative analysis is needed at this Stage.

Mathematics • Stage 5 (5.3 pathway)

Number and Algebra
Algebraic Techniques §

Outcomes
A student:

- uses and interprets formal definitions and generalisations when explaining solutions and/or conjectures MA5.3-1WM
- selects and applies appropriate algebraic techniques to operate with algebraic expressions MA5.3-5NA

Students:

Add and subtract algebraic fractions with binomial numerators and numerical denominators

- add and subtract algebraic fractions, including those with binomial numerators, eg $\frac{2x+5}{6}+\frac{x-4}{3}$, $\frac{x}{3}-\frac{x+1}{5}$

Expand binomial products and factorise monic quadratic expressions using a variety of strategies (ACMNA233)

- recognise and apply the special product, $(a-b)(a+b)=a^2-b^2$
  - recognise and name appropriate expressions as the 'difference of two squares' (Communicating) [L, CCT]
- recognise and apply the special products, $\left\{\begin{matrix}(a+b)^2=a^2+2ab+b^2 \\ (a-b)^2=a^2-2ab+b^2\end{matrix}\right\}$
  - recognise and name appropriate expressions as 'perfect squares' (Communicating) [L, CCT]
- use algebraic methods to expand a variety of binomial products, eg $(2y+1)^2$, $(3a-1)(3a+1)$, $(3x+1)(2-x)+2x+4$, $(x-y)^2-(x+y)^2$

Factorise monic and non-monic quadratic expressions and solve a wide range of quadratic equations derived from a variety of contexts (ACMNA269)

- factorise algebraic expressions by choosing an appropriate strategy [CCT]
  - common factors
  - difference of two squares
  - grouping in pairs for four-term expressions
  - trinomials
- use a variety of strategies to factorise expressions, eg $3d^3-3d$, $2a^2+12a+18$, $4x^2-20x+25$, $t^2-3t+st-3s$, $2a^2b-6ab-3a+9$ [CCT]
- factorise and simplify complex algebraic expressions involving algebraic fractions, eg $\frac{x^2+3x+2}{x+2}$, $\frac{4}{x^2+x}-\frac{3}{x^2-1}$, $\frac{3m-6}{4}\times\frac{8m}{m^2-2m}$, $\frac{4}{x^2-9}+\frac{2}{3x+9}$

Mathematics • Stage 5 (5.3 pathway)

Number and Algebra
Surds and Indices §

Outcomes
A student:

- uses and interprets formal definitions and generalisations when explaining solutions and/or conjectures — MA5.3-1WM
- generalises mathematical ideas and techniques to analyse and solve problems efficiently — MA5.3-2WM
- uses deductive reasoning in presenting arguments and formal proofs — MA5.3-3WM
- performs operations with surds and indices — MA5.3-6NA

Students:

Define rational and irrational numbers and perform operations with surds and fractional indices (ACMNA264)

- define real numbers; a real number is any number that can be represented by a point on the number line [L]
- define rational and irrational numbers; a rational number is the ratio $\frac{a}{b}$ of two integers *a* and *b* where $b \neq 0$. An irrational number is a real number that is not rational [L]
  - ‣ recognise that all rational and irrational numbers are real (Reasoning) [CCT]
  - ‣ explain why all integers and recurring decimals are rational numbers (Communicating, Reasoning) [L]
  - ‣ explain why rational numbers can be expressed in decimal form (Communicating, Reasoning) [L]
  - ‣ use a pair of compasses and a straight edge to construct simple rational numbers and surds on the number line (Problem Solving) [N]
- distinguish between rational and irrational numbers [CCT]
  - ‣ demonstrate that not all real numbers are rational (Problem Solving) [CCT]
- write recurring decimals in fraction form using calculator and non-calculator methods, eg $0.\dot{2}, 0.\dot{2}\dot{3}, 0.2\dot{3}$
  - ‣ justify why $0.\dot{9} = 1$ (Communicating, Reasoning) [CCT]
- demonstrate that $\sqrt{x}$ is undefined for $x < 0$ and $\sqrt{x} = 0$ for $x = 0$ [N]
- define $\sqrt{x}$ as the positive square root of *x* when $x > 0$ [L]
- use the following results for $x, y > 0$: $\left(\sqrt{x}\right)^2 = x = \sqrt{x^2}$

$$\sqrt{xy} = \sqrt{x} \times \sqrt{y}$$

$$\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$$

- apply the four operations of addition, subtraction, multiplication and division to simplify expressions involving surds

Mathematics • Stage 5 (5.3 pathway)

Number and Algebra
Surds and Indices §

- explain why a particular sentence is incorrect, eg explain why $\sqrt{3}+\sqrt{5}\neq\sqrt{8}$ (Communicating, Reasoning) [CCT]

- expand expressions involving surds, eg $\left(\sqrt{3}+\sqrt{5}\right)^2$, $\left(2-\sqrt{3}\right)\left(2+\sqrt{3}\right)$
  - connect operations with surds to algebraic techniques (Communicating) [CCT]
- rationalise the denominators of surds of the form $\dfrac{a\sqrt{b}}{c\sqrt{d}}$
  - investigate methods of rationalising surdic expressions with binomial denominators, making appropriate connections to algebraic techniques (Problem Solving) [CCT]
- recognise that a surd is an exact value that can be approximated by a rounded decimal
  - use surds to solve problems where a decimal answer is insufficient, eg find the exact perpendicular height of an equilateral triangle (Problem Solving) [N]
- establish that $\left(\sqrt{a}\right)^2=\sqrt{a}\times\sqrt{a}=\sqrt{a\times a}=\sqrt{a^2}=a$ [N]
- apply index laws to assist with the definition of the fractional index for the square root [L],

  eg $\left(\sqrt{a}\right)^2=a$

  and $\left(a^{\frac{1}{2}}\right)^2=a$

  $\therefore \quad \sqrt{a}=a^{\frac{1}{2}}$
  - explain why finding the square root of an expression is the same as raising the expression to the power of a half (Communicating, Reasoning) [CCT]
- use the index laws to demonstrate the reasonableness of the following definitions for fractional indices: $x^{\frac{1}{n}}=\sqrt[n]{x}$, $x^{\frac{m}{n}}=\sqrt[n]{x^m}$ [L]
- translate expressions in surd form to expressions in index form and vice versa
- use the $\boxed{x^{\frac{1}{y}}}$ or equivalent key on a scientific calculator
- evaluate numerical expressions involving fractional indices, eg $27^{\frac{2}{3}}$

Mathematics • Stage 5 (5.3 pathway)

Number and Algebra
Surds and Indices §

Background information

Operations with surds are applied when simplifying algebraic expressions.

Having expanded binomial products and rationalised denominators of surds of the form $\frac{a\sqrt{b}}{c\sqrt{d}}$, students could rationalise denominators of surds with binomial denominators.

Early Greek mathematicians believed that the length of any line would always be given by a rational number. This was proved to be false when Pythagoras and his followers found the length of the hypotenuse of an isosceles right-angled triangle with side length one unit.

Some students may enjoy a demonstration of the proof, by contradiction, that $\sqrt{2}$ is irrational.

Language

There is a need to emphasise how to read and articulate surds and fractional indices, eg $\sqrt{x}$ is ‘the square root of x’ or ‘root x’.

Mathematics • Stage 5 (5.3 pathway)

Number and Algebra
Equations §

Outcomes
A student:

- uses and interprets formal definitions and generalisations when explaining solutions and/or conjectures — MA5.3-1WM
- generalises mathematical ideas and techniques to analyse and solve problems efficiently — MA5.3-2WM
- uses deductive reasoning in presenting arguments and formal proofs — MA5.3-3WM
- solves complex linear, quadratic and simultaneous equations, and rearranges literal equations — MA5.3-7NA

Students:

Solve complex linear equations involving algebraic fractions

- solve a range of linear equations, including equations that involve two or more fractions, eg $\frac{2x-5}{3}-\frac{x+7}{5}=2$, $\frac{y-1}{4}-\frac{2y+3}{3}=\frac{1}{2}$

Factorise monic and non-monic quadratic expressions and solve a wide range of quadratic equations derived from a variety of contexts (ACMNA269)

- solve equations of the form $ax^2+bx+c=0$ using factors and by completing the square
- use the quadratic formula $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ to solve quadratic equations
- solve a variety of quadratic equations, eg $3x^2=4$, $x^2-8x-4=0$, $x(x-4)=4$, $(y-2)^2=9$
  - choose the most appropriate method to solve a particular quadratic equation (Problem Solving, Reasoning) [CCT]
- check the solutions of quadratic equations by substituting (Reasoning) [CCT]
- identify whether a given quadratic equation has no solution, one solution or two solutions [N]
  - predict the possible number of solutions for any quadratic equation (Communicating) [CCT]
  - connect the value of b^2-4ac to the number of possible solutions of $ax^2+bx+c=0$ and explain the significance of this connection (Communicating, Reasoning) [L]
- solve quadratic equations resulting from substitution into formulas
- create quadratic equations to solve a variety of problems and check solutions [N]
  - explain why one of the solutions to a quadratic equation generated from a word problem may not be a solution to the given problem (Communicating, Reasoning) [CCT]
- substitute a pronumeral to simplify higher order equations so they can be seen to belong to general categories and solve, eg substitute u for x^2 to solve $x^4-13x^2+36=0$

Solve literal equations

Mathematics • Stage 5 (5.3 pathway)

Number and Algebra
Equations §

- change the subject of formulas, including examples from other strands and other subjects, eg make *a* the subject of $v = u + at$; make *r* the subject of $\frac{1}{x} = \frac{1}{r} + \frac{1}{s}$; make *b* the subject of $x = \sqrt{b^2 - 4ac}$ [N]
  - determine restrictions on the values of variables implicit in the original formula and after rearrangement of the formula, eg by considering what restrictions there would be on the variables in the equation $Z = ax^2$ and what additional restrictions are assumed if the equation is rearranged to $x = \sqrt{\frac{Z}{a}}$ (Communicating, Reasoning) [N, CCT]

Solve simultaneous equations, using algebraic and graphical techniques

- use analytical methods to solve a variety of simultaneous equations, including those in which one equation is non-linear, eg $\begin{cases} 3x - 4y = 2 \\ 2x + y = 3 \end{cases}$, $\begin{cases} y = x^2 \\ y = x \end{cases}$, $\begin{cases} y = x^2 - x - 2 \\ y = x + 6 \end{cases}$, $\begin{cases} y = x + 5 \\ y = \dfrac{6}{x} \end{cases}$
  - choose an appropriate method to solve a pair of simultaneous equations (Problem Solving, Reasoning) [N]
  - compare the use of graphing software with algebraic methods in solving simultaneous equations (Communicating) [N, ICT, CCT]

Background information

The derivation of the quadratic formula can be demonstrated for more capable students.

Mathematics • Stage 5 (5.3 pathway)

Number and Algebra
Linear Relationships §

Outcomes
A student:
- uses and interprets formal definitions and generalisations when explaining solutions and/or conjectures MA5.3-1WM
- generalises mathematical ideas and techniques to analyse and solve problems efficiently MA5.3-2WM
- uses deductive reasoning in presenting arguments and formal proofs MA5.3-3WM
- uses formulas to find midpoint, gradient and distance, and applies standard forms of the equation of a straight line MA5.3-8NA

Students:

Find the midpoint and gradient of a line segment (interval) on the Cartesian plane using a range of strategies, including graphing software (ACMNA294)

- establish the formula for the midpoint, *M*, of the interval joining two points $\left(x_1, y_1\right)$ and $\left(x_2, y_2\right)$ on the number plane: $M(x, y)=\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ [N]
  - ‣ explain the meaning of each of the pronumerals in the formula for midpoint (Communicating) [CCT]
- use the formula to find the midpoint of the interval joining two points on the Cartesian plane
- use the relationship $\text{gradient}=\frac{\text{rise}}{\text{run}}$ to establish the formula for the gradient, *m*, of an interval joining two points $\left(x_1, y_1\right)$ and $\left(x_2, y_2\right)$ on the Cartesian plane: $m=\frac{y_2-y_1}{x_2-x_1}$ [N]
- use the formula to find the gradient of an interval joining two points on the Cartesian plane
  - ‣ explain why the formula $m=\frac{y_1-y_2}{x_1-x_2}$ gives the same solution as $m=\frac{y_2-y_1}{x_2-x_1}$ (Communicating, Reasoning) [CCT]

Find the distance between two points located on a Cartesian plane using a range of strategies, including graphing software (ACMNA214)

- use Pythagoras' theorem to establish the formula for the distance, *d*, between two points $\left(x_1, y_1\right)$ and $\left(x_2, y_2\right)$ on the Cartesian plane: $d=\sqrt{\left(x_2-x_1\right)^2+\left(y_2-y_1\right)^2}$ [N]
  - ‣ explain the meaning of each of the pronumerals in the formula for distance (Communicating) [CCT]
- use the formula to find the distance between two points on the Cartesian plane
  - ‣ explain why the formula $d=\sqrt{\left(x_1-x_2\right)^2+\left(y_1-y_2\right)^2}$ gives the same solution as $d=\sqrt{\left(x_2-x_1\right)^2+\left(y_2-y_1\right)^2}$ (Communicating, Reasoning) [CCT]

Mathematics • Stage 5 (5.3 pathway)

Number and Algebra
Linear Relationships §

Sketch linear graphs using the coordinates of two points and solve linear equations (ACMNA215)

- sketch the graph of a line by using its equation to find the x-and y-intercepts [N]

Solve problems using various standard forms of the equation of a straight line

- describe the equation of a line as the relationship between the x- and y-coordinates of any point on the line [N]
  - recognise from a list of equations those that result in straight line graphs (Communicating, Reasoning) [CCT]
- rearrange linear equations in gradient/intercept form into the general form $ax + by + c = 0$
- find the equation of a line passing through a point (x_1, y_1), with a given gradient m, using:
  $y - y_1 = m(x - x_1)$
  $y = mx + b$
- find the equation of a line passing through two points
- recognise and find the equation of a line in the general form: $ax + by + c = 0$

Solve problems involving parallel and perpendicular lines (ACMNA238)

- find the equation of a line that is parallel or perpendicular to a given line
- determine whether two given lines are perpendicular
  - use gradients to show that two given lines are perpendicular (Communicating, Problem Solving) [CCT]
- solve a variety of problems by applying coordinate geometry formulas and reasoning [N]
  - derive the formula for the distance between two points (Reasoning) [CCT]
  - show that two intervals with equal gradients and a common point form a straight line and use this to show that three points are collinear (Communicating, Reasoning) [N, CCT]
  - use coordinate geometry to investigate and describe the properties of triangles and quadrilaterals (Communicating, Problem Solving) [N, CCT]
  - use coordinate geometry to investigate the intersection of the perpendicular bisectors of the sides of acute-angled triangles (Problem Solving) [N, CCT]
  - show that four specified points form the vertices of particular quadrilaterals (Communicating, Problem Solving) [N, CCT]
  - prove that a particular triangle drawn on the Cartesian plane is right-angled (Communicating, Reasoning) [N, CCT]

Mathematics • Stage 5 (5.3 pathway)

Number and Algebra
Non-linear Relationships

Outcomes
A student:
- uses and interprets formal definitions and generalisations when explaining solutions and/or conjectures MA5.3-1WM
- uses deductive reasoning in presenting arguments and formal proofs MA5.3-3WM
- sketches and interprets a variety of non-linear relationships MA5.3-9NA

Students:

Describe, interpret and sketch parabolas, hyperbolas, circles and exponential functions and their transformations (ACMNA267)

- find *x*- and *y*-intercepts, where appropriate, for the graph of $y = ax^2 + bx + c$ given *a*, *b* and *c* [N]
- graph a variety of parabolas, including where the equation is given in the form $y = ax^2 + bx + c$ for various values of *a*, *b* and *c* [N]
  - use ICT to investigate and describe features of the graphs of parabolas given in the following forms for both positive and negative values of *a* and *k*
    eg $y = ax^2$
    $y = ax^2 + k$
    $y = (x + a)^2$
    $y = (x + a)^2 + k$
    (Communicating) [N, ICT, CCT]
  - describe features of a parabola by examining its equation (Communicating) [CCT]
- determine the equation of the axis of symmetry of a parabola using:
  - the midpoint of the interval joining the points at which the parabola cuts the *x*-axis [N, CCT]
  - the formula $x = -\dfrac{b}{2a}$ [N]
- find the coordinates of the vertex of a parabola using:
  - the midpoint of the interval joining the points at which the parabola cuts the x-axis and substitute to find the coordinates of its vertex [N]
  - the formula for the axis of symmetry to obtain the x-coordinate and substituting to obtain the y-coordinate of the vertex of a parabola [N]
  - completing the square on *x* in the equation of the parabola [N, CCT]
- identify and use features of parabolas and their equations to assist in sketching quadratic relationships, eg *x*- and *y*- intercepts, vertex, axis of symmetry and concavity [N]
- determine quadratic expressions to describe particular number patterns, eg generate the equation $y = x^2 + 1$ for the table [N]

x	0	1	2	3	4	5
y	1	2	5	10	17	26

- graph hyperbolic relationships of the form $y = \dfrac{k}{x}$ for integer values of *k* [N]

Mathematics • Stage 5 (5.3 pathway)

Number and Algebra
Non-linear Relationships

- describe the effect on the graph of $y = \frac{1}{x}$ of multiplying by different constants (Communicating) [N, CCT]
- explain what happens to the y-values of the points on the hyperbola $y = \frac{k}{x}$ as the x-values become very large or closer to zero (Communicating) [L, N, CCT]
- explain why it may be useful to choose small and large numbers when constructing a table of values for a hyperbola (Communicating, Reasoning) [CCT]

• graph a variety of hyperbolic curves, including where the equation is given in the form $y = \frac{k}{x} + c$ or $y = \frac{k}{x-b}$ for integer values of k, b and c [N]

- determine the equations of the asymptotes of a hyperbola in the form $y = \frac{k}{x} + c$ or $y = \frac{k}{x-b}$ (Problem Solving) [N, CCT]
- identify features of hyperbolas from their equations to assist in sketching their graphs, eg asymptotes, orientation, x- and/or y- intercepts where they exist (Problem Solving, Reasoning) [N, CCT]
- describe hyperbolas in terms of what happens to the y- values as x becomes very large or very small, whether there is a y- value for every x- value, and what occurs near or at $x = 0$ (Communicating, Reasoning) [N, CCT]

• recognise and describe the equations that represent circles with centre (h, k) and radius r [N, CCT]

- establish the equation of the circle centre (h, k), radius r, and graph equations of the form $(x-h)^2 + (y-k)^2 = r^2$ (Reasoning) [N, CCT]
- determine whether a particular point is inside, on, or outside a circle (Reasoning) [N, CCT]
- find the centre and radius of a circle whose equation is in the form $x^2 + fx + y^2 + gy = c$ by completing the square (Problem Solving)

• identify and namedifferent types of graphs from their equations, eg $(x-2)^2 + y^2 = 4$, $y = (x-2)^2 - 4$, $y = 4^x + 2$, $y = x^2 + 2x - 4$, $y = \frac{2}{x-4}$ [N, CCT]

- determine how to sketch a particular curve by determining its features from the equation (Problem Solving) [N, CCT]
- identify equations whose graph is symmetrical about the y- axis (Communicating, Reasoning) [N, CCT]

• determine a possible equation from a given graph and check using ICT [N, ICT]

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Number and Algebra
Non-linear Relationships

- ‣ compare and contrast different types of graphs and determine possible equations from key features, eg $y = 2$, $y = 2 - x$, $y = (x-2)^2$, $y = 2^x$, $(x-2)^2 + (y-2)^2 = 4$, $y = \frac{1}{x-2}$, $y = 2x^2$ (Communicating, Reasoning) [N, CCT, ICT]

- determine the points of intersection of a line with a parabola, circle or hyperbola, graphically and algebraically [N, ICT]
  - ‣ compare methods of finding points of intersection of curves and justify choice of method for a particular example (Communicating, Reasoning) [CCT]

Describe, interpret and sketch cubic functions, other curves and their transformations

- graph and compare features of the graphs of cubic equations of the forms
  $y = ax^3$
  $y = ax^3 + d$
  $y = a(x-r)(x-s)(x-t)$
  describing the effect on the graph of different values of a and d [L, N, CCT]
- graph a variety of equations of the form $y = ax^n$ for $n > 0$, describing the effect of n being odd or even on the shape of the curve [N, CCT]
- graph curves of the form $y = ax^n + k$ from curves of the form $y = ax^n$ by vertical transformations [N, CCT]
- graph curves of the form $y = a(x-r)^n$ from curves of the form $y = ax^n$ by using horizontal transformations [N, CCT]

Background information

This topic links to other subjects and real life examples of graphs, eg exponential graphs used for population growth in demographics, radioactive decay, town planning, etc.

This topic could provide opportunities for modelling. For example, the hyperbola $y = \frac{k}{x}$ for $x > 0$, models sharing a prize between x people, or length of a rectangle given area k and breadth x.

Mathematics • Stage 5 (5.3 pathway)

Number and Algebra
Polynomials #

Outcomes
A student:

- uses and interprets formal definitions and generalisations when explaining solutions and/or conjectures MA5.3-1WM
- generalises mathematical ideas and techniques to analyse and solve problems efficiently MA5.3-2WM
- uses deductive reasoning in presenting arguments and formal proofs MA5.3-3WM
- recognises, describes and sketches polynomials, and applies the factor and remainder theorems to solve problems MA5.3-10NA

Students:

Investigate the concept of a polynomial and apply the factor and remainder theorems to solve problems (ACMNA266)

- recognise a polynomial expression $a_nx^n + a_{n-1}x^{n-1} + ... + a_1x + a_0$ and use the terms degree, leading term, leading coefficient, constant term and monic polynomial appropriately [L]
- use the notation $P(x)$ for polynomials and $P(c)$ to indicate the value of $P(x)$ for $x = c$ [L]
- add and subtract polynomials and multiply polynomials by linear expressions
- divide polynomials by linear expressions to find the quotient and remainder, expressing the polynomial as the product of the linear expression and another polynomial plus a remainder [N], ie $P(x) = (x - a).Q(x) + c$
- verify the remainder theorem and use it to find factors of polynomials [N]
- use the factor theorem to factorise certain polynomials completely [N], ie if $(x - a)$ is a factor of $P(x)$, then $P(a) = 0$
- use the factor theorem and long division to find all zeros of a simple polynomial and hence solve $P(x) = 0$ (degree ≤ 4) [N]
- state the number of zeros that a polynomial of degree n can have [L, N]

Apply understanding of polynomials to sketch a range of curves and describe the features of these curves from their equation (ACMNA268)

- sketch the graphs of quadratic, cubic and quartic polynomials by factorising and finding the zeros [N]
  - recognise linear, quadratic and cubic expressions as examples of polynomials and relate sketching of these curves to factorising polynomials and finding the zeros (Reasoning) [CCT]
  - use digital technologies to graph polynomials of odd and even degree and investigate the relationship between the number of zeros and the degree of the polynomial (Communicating, Problem Solving) [N, ICT, CCT]
  - connect the roots of the equation to the x-intercepts and connect the constant term to the y-intercept (Communicating) [N, CCT]
  - determine the importance of the sign of the leading term of the polynomial on the behaviour of the curve as $x \to \pm\infty$ (Reasoning) [CCT]

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Number and Algebra
Polynomials #

- determine the effect of single, double and triple roots of a polynomial equation on the shape of the curve [CCT]
- use the leading term, the roots of the equation and the x- and y- intercepts to sketch polynomials [L, N]
  - describe the key features of a polynomial and draw its graph from the description (Communicating) [N, CCT]
- use the sketch of $y = P(x)$ to sketch $y = -P(x)$, $y = P(-x)$, $y = P(x) + c$, $y = aP(x)$ [N, CCT]
  - explain the similarities and differences between the graphs of two polynomials, such as $y = x^3 + x^2 + x$, $y = x^3 + x^2 + x + 1$ (Communicating) [N, CCT]

Mathematics • Stage 5 (5.3 pathway)

Number and Algebra
Logarithms #

Outcomes
A student:
- uses and interprets formal definitions and generalisations when explaining solutions and/or conjectures MA5.3-1WM
- uses deductive reasoning in presenting arguments and formal proofs MA5.3-3WM
- uses the definition of a logarithm to establish and apply the laws of logarithms MA5.3-11NA

Students:

Use the definition of a logarithm to establish and apply the laws of logarithms (ACMNA265)

- define logarithms; the logarithm of a number to any positive base is the index when the number is expressed as a power of the base, ie $a^x = y \Leftrightarrow \log_a y = x$ where $a > 0,\ y > 0$
- translate index statements into equivalent statements using logarithms [L, N],
eg $9 = 3^2,\ \therefore \log_3 9 = 2$

$$\frac{1}{2} = 2^{-1},\ \therefore \log_2 \frac{1}{2} = -1$$

$$4^{\frac{3}{2}} = 8,\quad \therefore \log_4 8 = \frac{3}{2}$$

- deduce the following laws of logarithms from the laws of indices [N]:

$$\log_a x + \log_a y = \log_a (xy)$$

$$\log_a x - \log_a y = \log_a \left(\frac{x}{y}\right)$$

$$\log_a x^n = n \log_a x$$

- establish and use the following results [N, CCT]:

$$\log_a a^x = x$$

$$\log_a a = 1$$

$$\log_a 1 = 0$$

$$\log_a \left(\frac{1}{x}\right) = -\log_a x$$

- apply the laws of logarithms to simplify simple expressions, eg $\log_2 8$, $\log_{81} 3$, $\log_{10} 25 + \log_{10} 4$, $3\log_{10} 2 + \log_{10}(12.5)$, $\log_2 18 - 2\log_2 3$ [N]
- simplify expressions using the laws of logarithms, eg simplify $5\log_a a - \log_a a^4$ [N]

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Number and Algebra
Logarithms #

- draw the graphs of the inverse functions $y = a^x$ and $y = \log_a x$ [N]
  - relate logarithms to practical scales that use indices, eg Richter, decibel and pH (Problem Solving) [N]
  - compare and contrast a set of exponential and logarithmic graphs drawn on the same axes, eg $y = 2^x$, $y = \log_2 x$, $y = 3^x$, $y = \log_3 x$ (Communicating, Reasoning) [N, CCT]

Solve simple exponential equations (ACMNA270)

- Solve simple equations that involve exponents or logarithms, eg $2^m = 8$, $4^{m+1} = \dfrac{1}{8\sqrt{2}}$, $\log_{27} 3 = x$, $\log_4 x = -2$ [N]

Background information

Logarithm tables were used to assist with calculations before the use of hand-held calculators. They converted multiplication and division to addition and subtraction, thus simplifying the calculations.

Language

Teachers need to emphasise the correct language used in connection with logarithms, eg $\log_a a^x = x$ is ‘log to the base *a* of *a* to the power of *x* equals *x*’.

Mathematics • Stage 5 (5.3 pathway)

Number and Algebra
Functions and Other Graphs #

Outcomes
A student:
- uses and interprets formal definitions and generalisations when explaining solutions and/or conjectures MA5.3-1WM
- uses deductive reasoning in presenting arguments and formal proofs MA5.3-3WM
- uses function notation to describe and sketch functions MA5.3-12NA

Students:

Describe, interpret and sketch functions

- define a function as a rule or relationship where for each input value there is only one output value, or that associates every member of one set with exactly one member of the second set [L, N]
- use the vertical line test on a graph to decide whether it represents a function [N]
  - explain whether a given graph represents a function (Communicating, Reasoning) [CCT]
  - decide whether straight line graphs always, sometimes or never represent a function (Reasoning) [CCT]
- use the notation $f(x)$ [L]
- use $f(c)$ notation to determine that value of $f(x)$ when $x = c$ [L]
- find the permissible x- and y-values for a variety of functions (including functions represented by straight lines, parabolas, exponentials and hyperbolas) [N]
- determine the inverse functions for a variety of functions and recognise their graphs as reflections in the line $y = x$ [L, N]
  - describe conditions for a function to have an inverse function (Communicating, Reasoning) [CCT]
  - recognise and describe the restrictions that need to be placed on quadratic functions so that they have an inverse function (Reasoning) [CCT]
- sketch the graphs of $y = f(x) + k$ and $y = f(x - a)$ given the graph of $y = f(x)$ [N, CCT]
  - sketch graphs to model relationships that occur in practical situations and explain the relationship between the variables represented in the graph (Communicating) [N]
  - consider a graph that represents a practical situation and explain the relationship between the two variables (Communicating) [N, CCT]

Mathematics • Stage 5 (5.3 pathway)

Measurement and Geometry
Area & Surface Area

Outcome
A student:
- uses and interprets formal definitions and generalisations when explaining solutions and/or conjectures MA5.3-1WM
- generalises mathematical ideas and techniques to analyse and solve problems efficiently MA5.3-2WM
- applies formulas to find the surface area of right pyramids, right cones, spheres and related composite solids MA5.3-13MG

Students:

Solve problems involving surface area and volume of right pyramids, right cones, spheres and related composite solids (ACMMG271)

- identify the perpendicular and slant height of pyramids and right cones [L]
- apply Pythagoras' theorem to find slant height, base length or perpendicular height of pyramids and right cones [N]
- devise and use methods to calculate the surface area of pyramids [N]
- develop and use the formula to calculate the surface area of cones
  $\text{Curved surface area of a cone} = \pi rl$
  where r is the length of the radius and l is the slant height [N, CCT]
- use the formula to calculate the surface area of spheres
  $\text{Surface area of a sphere} = 4\pi r^2$
  where r is the length of the radius [N]
- solve problems involving the surface area of solids [N]
  - find surface area of composite solids, eg a cone with a hemisphere on top (Problem Solving) [N]
  - find the dimensions of solids given their surface area by substitution into a formula to generate an equation (Problem Solving) [N]

Background information

Pythagoras' theorem is applied here to right-angled triangles in three-dimensional space.

The focus in this section is on right cones and right pyramids. Dealing with the oblique version of these objects is difficult and is mentioned only as a possible extension.

The area of the curved surface of a hemisphere is $2\pi r^2$, which is twice the area of its base. This may be a way of making the formula for the surface area of a sphere look reasonable to students. Deriving the relationship between the surface area and the volume of a sphere by dissection into very small pyramids may be an extension activity for some students. Similarly, some students may investigate as an extension, the surface area of a sphere by projection of very small squares onto a circumscribed cylinder.

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Measurement and Geometry
Volume

Outcome
A student:
- uses and interprets formal definitions and generalisations when explaining solutions and/or conjectures MA5.3-1WM
- generalises mathematical ideas and techniques to analyse and solve problems efficiently MA5.3-2WM
- uses deductive reasoning in presenting arguments and formal proofs MA5.3-3WM
- applies formulas to find the volume of right pyramids, right cones, spheres and related composite solids MA5.3-14MG

Students:

Solve problems involving surface area and volume of right pyramids, right cones, spheres and related composite solids (ACMMG271)

- develop and use the formula for the volume of pyramids and cones
  Volume of pyramid/cone $= \frac{1}{3}Ah$ where A is the base area and h is the perpendicular height [N, CCT]
  - recognise that a pyramid/cone has one-third the volume of a prism/cylinder with the same base and the same perpendicular height (Reasoning) [N, CCT]
  - deduce that the volume of a cone is given by $V = \frac{1}{3}\pi r^2 h$ (Reasoning) [N, CCT]
- use the formula to find the volume of spheres
  Volume of sphere $= \frac{4}{3}\pi r^3$ where r is the length of the radius [N]
- find the volume of composite solids that include right pyramids, right cones and hemispheres, eg find the volume of a cylinder with cone on top [N]
- solve problems relating to volume and/or capacity of right pyramids, cones and spheres [N]
  - apply Pythagoras' theorem as needed to calculate volumes of pyramids and cones (Problem Solving) [N]
  - find the dimensions of solids given their volume by substitution into a formula to generate an equation, eg find the length of the radius of a sphere given the volume (Problem Solving) [N]

Background information

The formulas for the volume of solids mentioned here depend only on the perpendicular height and apply equally well to the oblique case. The volume of oblique solids may be included as an extension for some students.

A more systematic development of the volume formulas for spheres, cones and pyramids can be given after integration is developed in Stage 6 (where the factor $\frac{1}{3}$ emerges essentially because the primitive of x^2 is $\frac{1}{3}x^3$).

At this stage, the relationship could be demonstrated by practical means, eg filling a pyramid with

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Measurement and Geometry
Volume

sand and pouring into a prism with the same base and perpendicular height and repeating until the prism is filled.

Some students may undertake the following exercise: visualise a cube of side length $2a$ dissected into six congruent pyramids with a common vertex at the centre of the cube, and hence prove that each of these pyramids has volume $\frac{4}{3}a^3$, which is $\frac{1}{3}$ of the enclosing rectangular prism.

The problem of finding the edge length of a cube that has twice the volume of another cube is called 'the duplication of the cube', and is one of three famous problems left unsolved by the ancient Greeks. It was proved in the 19th century that this cannot be done with straight edge and compasses, essentially because the cube root of 2 cannot be constructed on the number line.

Language

The difference between the meaning of 'slant height' and 'perpendicular height' of a prism, pyramid or cone should be made explicit for students.

Mathematics • Stage 5 (5.3 pathway)

Measurement and Geometry

Trigonometry and Pythagoras' Theorem §

Outcome

A student:

- uses and interprets formal definitions and generalisations when explaining solutions and/or conjectures MA5.3-1WM
- generalises mathematical ideas and techniques to analyse and solve problems efficiently MA5.3-2WM
- uses deductive reasoning in presenting arguments and formal proofs MA5.3-3WM
- applies Pythagoras' theorem, trigonometric relationships, the sine rule, the cosine rule and the area rule, to solve problems, including problems involving three dimensions MA5.3-15MG

Students:

Apply Pythagoras' theorem and trigonometry to solving three-dimensional problems in right-angled triangles (ACMMG276)

- solve problems involving the lengths of the edges and diagonals of rectangular prisms and other three-dimensional objects [N, CCT]
- use a given diagram to solve problems involving right-angled triangles in three dimensions [N]
  - check the reasonableness of answers to trigonometry problems in three dimensions (Problem Solving) [N, CCT]
- draw diagrams and use them to solve word problems involving right-angled triangles in three dimensions, including using bearings and angles of elevation or depression, eg 'From a point, *A,* due south of a flagpole 100 metre tall on level ground, the angle of elevation of the top of the flagpole is 35°. The top of the same flagpole is observed with angle of elevation 22° from a point, *B*, due east of the flagpole. What is the distance from *A* to *B*?' [N, CCT]
  - check the reasonableness of answers to trigonometry problems in three dimensions (Problem Solving) [N, CCT]

Use the unit circle to define trigonometric functions, and graph them with and without the use of digital technologies (ACMMG274)

- prove that the tangent ratio can be expressed as a ratio of the sine and cosine ratios $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ [N, CCT]
- use the unit circle and ICT to investigate the sine, cosine and tangent curves for at least $0° \le A \le 360°$ and sketch the results [L, N, CCT]
  - compare features of these trigonometric curves including periodicity and symmetry (Communicating) [N, CCT]
  - describe how trigonometric ratios change as the angle increases from $0°$ to $360°$ (Communicating) [N, CCT]

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Measurement and Geometry

Trigonometry and Pythagoras' Theorem §

- use the unit circle or graphs of trigonometric functions to establish and use the following relationships for obtuse angles, where $0° \le A \le 90°$:

$$\sin A = \sin(180° - A)$$
$$\cos A = -\cos(180° - A) \text{ [CCT]}$$
$$\tan A = -\tan(180° - A)$$

  - recognise that if $\sin A \ge 0$ then there are two possible values for *A*, given $0° \le A \le 180°$ (Reasoning) [N, CCT]
- find the angle of inclination, θ, of a line in the coordinate plane by establishing and using the relationship gradient = $\tan\theta$ [N, CCT]

Solve simple trigonometric equations (ACMMG275)

- determine and use exact sine, cosine and tangent ratios for angles of 30°, 45°, 60° [L, N]
  - solve problems in right-angled triangles using exact trigonometric ratios for 30°, 45° and 60° (Problem Solving) [N]
- prove and use the relationship between the sine and cosine ratios of complementary angles in right-angled triangles

$$\sin A = \cos(90° - A) \text{ [CCT]}$$
$$\cos A = \sin(90° - A)$$

- find the possible acute and/or obtuse angles, given a trigonometric ratio [N]

Establish the sine, cosine and area rules for any triangle and solve related problems (ACMMG273)

- prove the sine rule:
  in a given triangle ABC, the ratio of a side to the sine of the opposite angle is a constant.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \text{ [L, CCT]}$$

- use the sine rule to find unknown sides and angles of a triangle, including in problems where there are two possible solutions for an angle [N]
  - recognise that if given two sides and a non-included angle then two triangles may result, leading to two solutions when the sine rule is applied (Reasoning) [N, CCT]
- prove the cosine rule:
  in a given triangle *ABC*, $a^2 = b^2 + c^2 - 2bc\cos A$ [L, CCT]

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

- use the cosine rule to find unknown sides and angles of a triangle [N]
- prove and use the area rule to find the area of a triangle:
  in a given triangle *ABC*, $\text{Area of triangle} = \frac{1}{2}ab\sin C$ [L, CCT]
- select and apply the appropriate rule to find unknowns in non-right-angled triangles [N, CCT]
  - explain what happens if the sine, cosine and area rules are applied in right-angled triangles (Communicating, Reasoning) [N, CCT]

Mathematics • Stage 5 (5.3 pathway)

Measurement and Geometry

Trigonometry and Pythagoras' Theorem §

- draw diagrams and use them to solve word problems that involve non-right-angled triangles [N, CCT]
  - use appropriate trigonometric ratios and formulas to solve two-dimensional trigonometric problems that require the use of more than one triangle, where the diagram is provided and where a verbal description is given (Problem Solving) [N, CCT]
  - solve problems, including practical problems, involving the sine and cosine rules and the area rule, eg problems related to surveying or orienteering (Problem Solving) [N]

Background information

Pythagoras' theorem is applied here to right-angled triangles in three-dimensional space.

The trigonometric functions here could be redefined for the general angle using a circle in the coordinate plane. This allows the sine and cosine functions to be plotted for a full revolution and beyond so that their wave nature becomes clear. The intention, however, of this section is for students to become confident using the sine and cosine rules and the area rule in practical situations. For many students it is therefore more appropriate to justify the extension of the trigonometric functions to obtuse angles only, either by plotting the graphs and continuing them in the obvious way, or by taking the identities for $180° - \theta$ as definitions. Whatever is done, experimentation with the calculator should be used to confirm this extension.

The graphs of the trigonometric functions mark the transition from understanding trigonometry as the study of lengths and angles in triangles (as the word trigonometry implies) to the study of waves, as will be developed in the Stage 6 calculus courses. Waves are fundamental to a vast range of physical and practical phenomena, like light waves and all other electromagnetic waves, and to periodic phenomena like daily temperatures and fluctuating sales over the year, and the major importance of trigonometry lies in the study of these waves.

The formula gradient = $\tan\theta$ is a formula for gradient in the coordinate plane.

Students are not expected to reproduce proofs of the sine, cosine and area rules. The cosine rule is a generalisation of Pythagoras' theorem. The sine rule is linked to the circumcircle and to circle geometry.

Students at this level should realise that when the triangle is right-angled, the cosine rule becomes Pythagoras' theorem, the area formula becomes the simple 'half base times perpendicular height' formula, and the sine rule becomes a simple application of the sine function in a right-angled triangle.

Mathematics • Stage 5 (5.3 pathway)

Measurement and Geometry
Properties of Geometrical Figures §

Outcome
A student:
- uses and interprets formal definitions and generalisations when explaining solutions and/or conjectures MA5.3-1WM
- generalises mathematical ideas and techniques to analyse and solve problems efficiently MA5.3-2WM
- uses deductive reasoning in presenting arguments and formal proofs MA5.3-3WM
- proves triangles are similar, and uses formal geometric reasoning to establish properties of triangles and quadrilaterals MA5.3-16MG

Students:

Formulate proofs involving congruent triangles and angle properties (ACMMG243)

- construct and write geometrical arguments to prove a general geometrical result, giving reasons at each step of the argument, eg prove that the angle in a semicircle is a right angle [CCT]

Apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes (ACMMG244)

- write formal proofs of similarity of triangles in the standard four- or five-line format, preserving the matching order of vertices, identifying the similarity factor when appropriate, and drawing relevant conclusions from this similarity [L, CCT]
  - ‣ prove that the interval joining the midpoints of two sides of a triangle is parallel to the third side and half its length, and its converse (Communicating, Problem Solving) [CCT]
- establish and apply the fact that in two similar figures with similarity ratio 1: k [N, CCT]
  - – matching angles have the same size
  - – matching intervals are in the ratio 1: k
  - – matching areas are in the ratio 1: k^2
  - – matching volumes are in the ratio 1: k^3
  - ‣ solve problems involving the similarity ratio and areas and volumes (Problem Solving) [N]
- state a definition as the minimum amount of information needed to identify a particular figure [L, CCT]
- prove properties of isosceles and equilateral triangles and special quadrilaterals using the formal definitions of shapes [L, CCT]
  Definitions:
  - – a scalene triangle is a triangle with no two sides equal in length
  - – an isosceles triangle is a triangle with two sides equal in length
  - – an equilateral triangle is a triangle with all sides equal in length
  - – a trapezium is a quadrilateral with at least one pair of opposite sides parallel
  - – a parallelogram is a quadrilateral with both pairs of opposite sides parallel
  - – a rhombus is a parallelogram with two adjacent sides equal in length
  - – a rectangle is a parallelogram with one angle a right angle
  - – a square is a rectangle with two adjacent sides equal.
  - ‣ use dynamic geometry software to investigate and test conjectures about geometrical figures (Problem Solving, Reasoning) [ICT, CCT]

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Measurement and Geometry
Properties of Geometrical Figures §

- prove and apply theorems and properties related to triangles and quadrilaterals such as:
  - the sum of the interior angles of a triangle is 180°
  - the exterior angle of a triangle is equal to the sum of the two interior opposite angles
  - if two sides of a triangle are equal, then the angles opposite those sides are equal
  - conversely, if two angles of a triangle are equal, then the sides opposite those angles are equal
  - each angle of an equilateral triangle is equal to 60°
  - the sum of the interior angles of a quadrilateral is 360°
  - the opposite angles of a parallelogram are equal
  - the opposite sides of a parallelogram are equal
  - the diagonals of a parallelogram bisect each other
  - the diagonals of a rhombus bisect each other at right angles
  - the diagonals of a rhombus bisect the vertex angles through which they pass
  - the diagonals of a rectangle are equal [N, CCT]

  ‣ reason that any result proven for a parallelogram would also hold for a rectangle (Reasoning) [CCT]

  ‣ give reasons why a square is a rhombus, but a rhombus is not necessarily a square (Communicating, Reasoning) [CCT]

  ‣ use a diagram or flow chart to show the relationships between different quadrilaterals (Communicating) [CCT]

- prove and apply tests for quadrilaterals:
  - if both pairs of opposite angles of a quadrilateral are equal, then it is a parallelogram
  - if both pairs of opposite sides of a quadrilateral are equal, then it is a parallelogram
  - if all sides of a quadrilateral are equal, then it is a rhombus [CCT]

- solve numerical and non-numerical problems in Euclidean geometry based on known assumptions and proven theorems [N, CCT]

  ‣ state possible converses of known results, and examine whether or not they are also true (Communicating, Reasoning) [CCT]

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Measurement and Geometry
Properties of Geometrical Figures §

Background information

In Circle Geometry, similarity of triangles is used to prove further theorems on intersecting chords, secants and tangents.

Attention should be given to the logical sequence of theorems and to the types of arguments used. Memorisation of proofs is not intended. Every theorem presented could be preceded by a straight-edge-and-compasses construction to confirm the theorem, before it is proven in a manner appropriate to the student's work level by way of an exercise or an investigation.

In Euclidean geometry, congruence is the method used to construct symmetry arguments. It is often helpful to see exactly what transformation, or sequence of transformations, will map one triangle into a congruent triangle. For example, the proof that the opposite sides of a parallelogram are equal involves constructing a diagonal and proving that the resulting triangles are congruent – these two triangles can be transformed into each other by a rotation of 180º about the midpoint of the diagonal.

In Einstein's general theory of relativity, three-dimensional space is curved and, as a result, the sum of the angles of a physical triangle of cosmological proportions is not 180º. Abstract geometries of this nature were developed by Gauss, Bolyai, Lobachevsky and Riemann, others in the early 19th century, amid suspicions that Euclidean geometry may not be the correct description of physical space.

The Elements of Euclid (c325−265 BC) gives an account of geometry written almost entirely as a sequence of axioms, definitions, theorems and proofs. Its methods have had an enormous influence on mathematics. Students could read some of Book 1 for a far more systematic account of the geometry of triangles and quadrilaterals.

Mathematics • Stage 5 (5.3 pathway)

Measurement and Geometry
Circle Geometry #

Outcome
A student:
- uses and interprets formal definitions and generalisations when explaining solutions and/or conjectures MA5.3-1WM
- generalises mathematical ideas and techniques to analyse and solve problems efficiently MA5.3-2WM
- uses deductive reasoning in presenting arguments and formal proofs MA5.3-3WM
- applies deductive reasoning to prove circle theorems and to solve related problems MA5.3-17MG

Students:

Prove and apply angle and chord properties of circles (ACMMG272)

- identify and name parts of a circle (centre, radius, diameter, circumference, sector, arc, chord, secant, tangent, segment, semicircle) [L]
- use terminology associated with angles in circles such as subtend, standing on the same arc, angle at the centre, angle at the circumference, angle in a segment [L]
- identify the arc on which an angle at the centre or circumference stands [CCT]
- demonstrate that at any point on a circle there is a unique tangent to the circle, and that this tangent is perpendicular to the radius at the point of contact [CCT]
- prove the following chord properties of circles:
  - chords of equal length in a circle subtend equal angles at the centre and are equidistant from the centre
  - the perpendicular from the centre of a circle to a chord bisects the chord
  - conversely, the line from the centre of a circle to the midpoint of a chord is perpendicular to the chord
  - the perpendicular bisector of a chord of a circle passes through the centre
  - given any three non-collinear points, the point of intersection of the perpendicular bisectors of any two sides of the triangle formed by the three points is the centre of the circle through all three points
  - when two circles intersect, the line joining their centres bisects their common chord at right angles [CCT]
  - ‣ use dynamic geometry software to investigate chord properties of circles (Problem Solving, Reasoning) [ICT, CCT]
- prove the following angle properties involving circles:
  - the angle at the centre of a circle is twice the angle at the circumference standing on the same arc
  - the angle in a semicircle is a right angle
  - angles at the circumference, standing on the same arc, are equal
  - the opposite angles of cyclic quadrilaterals are supplementary
  - an exterior angle at a vertex of a cyclic quadrilateral is equal to the interior opposite angle [CCT]
  - ‣ use dynamic geometry software to investigate angle properties of circles (Communicating, Problem Solving, Reasoning) [ICT, CCT]
  - ‣ apply circle theorems to prove that the angle in a semicircle is a right angle (Communicating, Problem Solving, Reasoning) [CCT]

Mathematics • Stage 5 (5.3 pathway)

Measurement and Geometry
Circle Geometry #

- prove the following tangent and secant properties involving circles:
  - the two tangents drawn to a circle from an external point are equal in length
  - the angle between a tangent and a chord drawn to the point of contact is equal to the angle in the alternate segment
  - when two circles touch, their centres and the point of contact are collinear
  - the products of the intercepts of two intersecting chords of a circle are equal
  - the products of the intercepts of two intersecting secants to a circle from an external point are equal
  - the square of a tangent to a circle from an external point equals the product of the intercepts of any secants from the point [CCT]
    - ‣ use dynamic geometry software to investigate tangent properties of circles (Communicating, Problem Solving, Reasoning) [ICT, CCT]
- apply circle theorems to find unknown angles and sides in diagrams [N, CCT]

Background information

As well as solving arithmetic and algebraic problems in circle geometry, students should be able to reason deductively within more theoretical arguments. Diagrams would normally be given to students, with the important information labelled on the diagram to aid reasoning. Students would sometimes need to produce a clear diagram from a set of instructions.

Attention should be given to the logical sequence of theorems and to the types of arguments used. Memorisation of proofs is not intended. Ideally, every theorem presented should be preceded by a straight-edge-and-compasses construction to confirm the theorem, before it is proven in a manner appropriate to the student's work level by way of an exercise or an investigation.

The tangent-and-radius theorem is difficult to justify at this stage and is probably better taken as an assumption as indicated above.

Circle Geometry may be extended to examining the converse of some of the theorems related to cyclic quadrilaterals, leading to a series of conditions for points to be concyclic. However, students may find these results difficult to prove and apply.

The angle in a semicircle theorem is also called Thales' theorem because it was traditionally ascribed to Thales (c624–548 BC) by the ancient Greeks, who reported that it was the first theorem ever proven in mathematics.

Mathematics • Stage 5 (5.3 pathway)

Measurement and Geometry
Area & Surface Area

Outcome
A student:
- uses and interprets formal definitions and generalisations when explaining solutions and/or conjectures MA5.3-1WM
- generalises mathematical ideas and techniques to analyse and solve problems efficiently MA5.3-2WM
- applies formulas to find the surface area of right pyramids, right cones, spheres and related composite solids MA5.3-13MG

Students:

Solve problems involving surface area and volume of right pyramids, right cones, spheres and related composite solids (ACMMG271)

- identify the perpendicular and slant height of pyramids and right cones [L]
- apply Pythagoras' theorem to find slant height, base length or perpendicular height of pyramids and right cones [N]
- devise and use methods to calculate the surface area of pyramids [N]
- develop and use the formula to calculate the surface area of cones
  $\text{Curved surface area of a cone} = \pi rl$
  where r is the length of the radius and l is the slant height [N, CCT]
- use the formula to calculate the surface area of spheres
  $\text{Surface area of a sphere} = 4\pi r^2$
  where r is the length of the radius [N]
- solve problems involving the surface area of solids [N]
  - find surface area of composite solids, eg a cone with a hemisphere on top (Problem Solving) [N]
  - find the dimensions of solids given their surface area by substitution into a formula to generate an equation (Problem Solving) [N]

Background information

Pythagoras' theorem is applied here to right-angled triangles in three-dimensional space.

The focus in this section is on right cones and right pyramids. Dealing with the oblique version of these objects is difficult and is mentioned only as a possible extension.

The area of the curved surface of a hemisphere is $2\pi r^2$, which is twice the area of its base. This may be a way of making the formula for the surface area of a sphere look reasonable to students. Deriving the relationship between the surface area and the volume of a sphere by dissection into very small pyramids may be an extension activity for some students. Similarly, some students may investigate as an extension, the surface area of a sphere by projection of very small squares onto a circumscribed cylinder.

Mathematics • Stage 5 (5.3 pathway)

Measurement and Geometry
Volume

Outcome
A student:
- uses and interprets formal definitions and generalisations when explaining solutions and/or conjectures MA5.3-1WM
- generalises mathematical ideas and techniques to analyse and solve problems efficiently MA5.3-2WM
- uses deductive reasoning in presenting arguments and formal proofs MA5.3-3WM
- applies formulas to find the volume of right pyramids, right cones, spheres and related composite solids MA5.3-14MG

Students:

Solve problems involving surface area and volume of right pyramids, right cones, spheres and related composite solids (ACMMG271)

- develop and use the formula for the volume of pyramids and cones

  $\text{Volume of pyramid/cone} = \frac{1}{3}Ah$ where A is the base area and h is the perpendicular height [N, CCT]
  - recognise that a pyramid/cone has one-third the volume of a prism/cylinder with the same base and the same perpendicular height (Reasoning) [N, CCT]
  - deduce that the volume of a cone is given by $V = \frac{1}{3}\pi r^2 h$ (Reasoning) [N, CCT]
- use the formula to find the volume of spheres

  $\text{Volume of sphere} = \frac{4}{3}\pi r^3$ where r is the length of the radius [N]
- find the volume of composite solids that include right pyramids, right cones and hemispheres, eg find the volume of a cylinder with cone on top [N]
- solve problems relating to volume and/or capacity of right pyramids, cones and spheres [N]
  - apply Pythagoras' theorem as needed to calculate volumes of pyramids and cones (Problem Solving) [N]
  - find the dimensions of solids given their volume by substitution into a formula to generate an equation, eg find the length of the radius of a sphere given the volume (Problem Solving) [N]

Background information

The formulas for the volume of solids mentioned here depend only on the perpendicular height and apply equally well to the oblique case. The volume of oblique solids may be included as an extension for some students.

A more systematic development of the volume formulas for spheres, cones and pyramids can be given after integration is developed in Stage 6 (where the factor $\frac{1}{3}$ emerges essentially because the primitive of x^2 is $\frac{1}{3}x^3$).

At this stage, the relationship could be demonstrated by practical means, eg filling a pyramid with

Mathematics • Stage 5 (5.3 pathway)

Measurement and Geometry
Volume

sand and pouring into a prism with the same base and perpendicular height and repeating until the prism is filled.

Some students may undertake the following exercise: visualise a cube of side length $2a$ dissected into six congruent pyramids with a common vertex at the centre of the cube, and hence prove that each of these pyramids has volume $\frac{4}{3}a^3$, which is $\frac{1}{3}$ of the enclosing rectangular prism.

The problem of finding the edge length of a cube that has twice the volume of another cube is called 'the duplication of the cube', and is one of three famous problems left unsolved by the ancient Greeks. It was proved in the 19th century that this cannot be done with straight edge and compasses, essentially because the cube root of 2 cannot be constructed on the number line.

Language

The difference between the meaning of 'slant height' and 'perpendicular height' of a prism, pyramid or cone should be made explicit for students.

Mathematics • Stage 5 (5.3 pathway)

Measurement and Geometry

Trigonometry and Pythagoras' Theorem §

Outcome

A student:

- uses and interprets formal definitions and generalisations when explaining solutions and/or conjectures MA5.3-1WM
- generalises mathematical ideas and techniques to analyse and solve problems efficiently MA5.3-2WM
- uses deductive reasoning in presenting arguments and formal proofs MA5.3-3WM
- applies Pythagoras' theorem, trigonometric relationships, the sine rule, the cosine rule and the area rule, to solve problems, including problems involving three dimensions MA5.3-15MG

Students:

Apply Pythagoras' theorem and trigonometry to solving three-dimensional problems in right-angled triangles (ACMMG276)

- solve problems involving the lengths of the edges and diagonals of rectangular prisms and other three-dimensional objects [N, CCT]
- use a given diagram to solve problems involving right-angled triangles in three dimensions [N]
  - check the reasonableness of answers to trigonometry problems in three dimensions (Problem Solving) [N, CCT]
- draw diagrams and use them to solve word problems involving right-angled triangles in three dimensions, including using bearings and angles of elevation or depression, eg 'From a point, *A,* due south of a flagpole 100 metre tall on level ground, the angle of elevation of the top of the flagpole is 35°. The top of the same flagpole is observed with angle of elevation 22° from a point, *B*, due east of the flagpole. What is the distance from *A* to *B*?' [N, CCT]
  - check the reasonableness of answers to trigonometry problems in three dimensions (Problem Solving) [N, CCT]

Use the unit circle to define trigonometric functions, and graph them with and without the use of digital technologies (ACMMG274)

- prove that the tangent ratio can be expressed as a ratio of the sine and cosine ratios $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ [N, CCT]
- use the unit circle and ICT to investigate the sine, cosine and tangent curves for at least $0° \le A \le 360°$ and sketch the results [L, N, CCT]
  - compare features of these trigonometric curves including periodicity and symmetry (Communicating) [N, CCT]
  - describe how trigonometric ratios change as the angle increases from $0°$ to $360°$ (Communicating) [N, CCT]

Mathematics • Stage 5 (5.3 pathway)

Measurement and Geometry
Trigonometry and Pythagoras' Theorem §

- use the unit circle or graphs of trigonometric functions to establish and use the following relationships for obtuse angles, where $0° \le A \le 90°$:
  $\sin A = \sin(180° - A)$
  $\cos A = -\cos(180° - A)$ [CCT]
  $\tan A = -\tan(180° - A)$
  - recognise that if $\sin A \ge 0$ then there are two possible values for *A*, given $0° \le A \le 180°$ (Reasoning) [N, CCT]
- find the angle of inclination, θ, of a line in the coordinate plane by establishing and using the relationship gradient = $\tan\theta$ [N, CCT]

Solve simple trigonometric equations (ACMMG275)

- determine and use exact sine, cosine and tangent ratios for angles of 30°, 45°, 60° [L, N]
  - solve problems in right-angled triangles using exact trigonometric ratios for 30°, 45° and 60° (Problem Solving) [N]
- prove and use the relationship between the sine and cosine ratios of complementary angles in right-angled triangles
  $\sin A = \cos(90° - A)$ [CCT]
  $\cos A = \sin(90° - A)$
- find the possible acute and/or obtuse angles, given a trigonometric ratio [N]

Establish the sine, cosine and area rules for any triangle and solve related problems (ACMMG273)

- prove the sine rule:
  in a given triangle ABC, the ratio of a side to the sine of the opposite angle is a constant.
  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ [L, CCT]
- use the sine rule to find unknown sides and angles of a triangle, including in problems where there are two possible solutions for an angle [N]
  - recognise that if given two sides and a non-included angle then two triangles may result, leading to two solutions when the sine rule is applied (Reasoning) [N, CCT]
- prove the cosine rule:
  in a given triangle *ABC*, $a^2 = b^2 + c^2 - 2bc\cos A$ [L, CCT]
  $$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$
- use the cosine rule to find unknown sides and angles of a triangle [N]
- prove and use the area rule to find the area of a triangle:
  in a given triangle *ABC*, $\text{Area of triangle} = \frac{1}{2}ab\sin C$ [L, CCT]
- select and apply the appropriate rule to find unknowns in non-right-angled triangles [N, CCT]
  - explain what happens if the sine, cosine and area rules are applied in right-angled triangles (Communicating, Reasoning) [N, CCT]

Mathematics • Stage 5 (5.3 pathway)

Measurement and Geometry
Trigonometry and Pythagoras' Theorem §

- draw diagrams and use them to solve word problems that involve non-right-angled triangles [N, CCT]
  - use appropriate trigonometric ratios and formulas to solve two-dimensional trigonometric problems that require the use of more than one triangle, where the diagram is provided and where a verbal description is given (Problem Solving) [N, CCT]
  - solve problems, including practical problems, involving the sine and cosine rules and the area rule, eg problems related to surveying or orienteering (Problem Solving) [N]

Background information

Pythagoras' theorem is applied here to right-angled triangles in three-dimensional space.

The trigonometric functions here could be redefined for the general angle using a circle in the coordinate plane. This allows the sine and cosine functions to be plotted for a full revolution and beyond so that their wave nature becomes clear. The intention, however, of this section is for students to become confident using the sine and cosine rules and the area rule in practical situations. For many students it is therefore more appropriate to justify the extension of the trigonometric functions to obtuse angles only, either by plotting the graphs and continuing them in the obvious way, or by taking the identities for $180° - \theta$ as definitions. Whatever is done, experimentation with the calculator should be used to confirm this extension.

The graphs of the trigonometric functions mark the transition from understanding trigonometry as the study of lengths and angles in triangles (as the word trigonometry implies) to the study of waves, as will be developed in the Stage 6 calculus courses. Waves are fundamental to a vast range of physical and practical phenomena, like light waves and all other electromagnetic waves, and to periodic phenomena like daily temperatures and fluctuating sales over the year, and the major importance of trigonometry lies in the study of these waves.

The formula gradient = $\tan\theta$ is a formula for gradient in the coordinate plane.

Students are not expected to reproduce proofs of the sine, cosine and area rules. The cosine rule is a generalisation of Pythagoras' theorem. The sine rule is linked to the circumcircle and to circle geometry.

Students at this level should realise that when the triangle is right-angled, the cosine rule becomes Pythagoras' theorem, the area formula becomes the simple 'half base times perpendicular height' formula, and the sine rule becomes a simple application of the sine function in a right-angled triangle.

Mathematics • Stage 5 (5.3 pathway)

Measurement and Geometry
Properties of Geometrical Figures §

Outcome
A student:

- uses and interprets formal definitions and generalisations when explaining solutions and/or conjectures MA5.3-1WM
- generalises mathematical ideas and techniques to analyse and solve problems efficiently MA5.3-2WM
- uses deductive reasoning in presenting arguments and formal proofs MA5.3-3WM
- proves triangles are similar, and uses formal geometric reasoning to establish properties of triangles and quadrilaterals MA5.3-16MG

Students:

Formulate proofs involving congruent triangles and angle properties (ACMMG243)

- construct and write geometrical arguments to prove a general geometrical result, giving reasons at each step of the argument, eg prove that the angle in a semicircle is a right angle [CCT]

Apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes (ACMMG244)

- write formal proofs of similarity of triangles in the standard four- or five-line format, preserving the matching order of vertices, identifying the similarity factor when appropriate, and drawing relevant conclusions from this similarity [L, CCT]
  - ‣ prove that the interval joining the midpoints of two sides of a triangle is parallel to the third side and half its length, and its converse (Communicating, Problem Solving) [CCT]
- establish and apply the fact that in two similar figures with similarity ratio 1: k [N, CCT]
  - matching angles have the same size
  - matching intervals are in the ratio 1: k
  - matching areas are in the ratio 1: k^2
  - matching volumes are in the ratio 1: k^3
  - ‣ solve problems involving the similarity ratio and areas and volumes (Problem Solving) [N]
- state a definition as the minimum amount of information needed to identify a particular figure [L, CCT]
- prove properties of isosceles and equilateral triangles and special quadrilaterals using the formal definitions of shapes [L, CCT]
  Definitions:
  - a scalene triangle is a triangle with no two sides equal in length
  - an isosceles triangle is a triangle with two sides equal in length
  - an equilateral triangle is a triangle with all sides equal in length
  - a trapezium is a quadrilateral with at least one pair of opposite sides parallel
  - a parallelogram is a quadrilateral with both pairs of opposite sides parallel
  - a rhombus is a parallelogram with two adjacent sides equal in length
  - a rectangle is a parallelogram with one angle a right angle
  - a square is a rectangle with two adjacent sides equal.
  - ‣ use dynamic geometry software to investigate and test conjectures about geometrical figures (Problem Solving, Reasoning) [ICT, CCT]

Mathematics • Stage 5 (5.3 pathway)

Measurement and Geometry
Properties of Geometrical Figures §

- prove and apply theorems and properties related to triangles and quadrilaterals such as:
  - the sum of the interior angles of a triangle is 180°
  - the exterior angle of a triangle is equal to the sum of the two interior opposite angles
  - if two sides of a triangle are equal, then the angles opposite those sides are equal
  - conversely, if two angles of a triangle are equal, then the sides opposite those angles are equal
  - each angle of an equilateral triangle is equal to 60°
  - the sum of the interior angles of a quadrilateral is 360°
  - the opposite angles of a parallelogram are equal
  - the opposite sides of a parallelogram are equal
  - the diagonals of a parallelogram bisect each other
  - the diagonals of a rhombus bisect each other at right angles
  - the diagonals of a rhombus bisect the vertex angles through which they pass
  - the diagonals of a rectangle are equal [N, CCT]

  ‣ reason that any result proven for a parallelogram would also hold for a rectangle (Reasoning) [CCT]

  ‣ give reasons why a square is a rhombus, but a rhombus is not necessarily a square (Communicating, Reasoning) [CCT]

  ‣ use a diagram or flow chart to show the relationships between different quadrilaterals (Communicating) [CCT]

- prove and apply tests for quadrilaterals:
  - if both pairs of opposite angles of a quadrilateral are equal, then it is a parallelogram
  - if both pairs of opposite sides of a quadrilateral are equal, then it is a parallelogram
  - if all sides of a quadrilateral are equal, then it is a rhombus [CCT]

- solve numerical and non-numerical problems in Euclidean geometry based on known assumptions and proven theorems [N, CCT]

  ‣ state possible converses of known results, and examine whether or not they are also true (Communicating, Reasoning) [CCT]

Mathematics • Stage 5 (5.3 pathway)

Measurement and Geometry
Properties of Geometrical Figures §

Background information

In Circle Geometry, similarity of triangles is used to prove further theorems on intersecting chords, secants and tangents.

Attention should be given to the logical sequence of theorems and to the types of arguments used. Memorisation of proofs is not intended. Every theorem presented could be preceded by a straight-edge-and-compasses construction to confirm the theorem, before it is proven in a manner appropriate to the student's work level by way of an exercise or an investigation.

In Euclidean geometry, congruence is the method used to construct symmetry arguments. It is often helpful to see exactly what transformation, or sequence of transformations, will map one triangle into a congruent triangle. For example, the proof that the opposite sides of a parallelogram are equal involves constructing a diagonal and proving that the resulting triangles are congruent – these two triangles can be transformed into each other by a rotation of 180º about the midpoint of the diagonal.

In Einstein's general theory of relativity, three-dimensional space is curved and, as a result, the sum of the angles of a physical triangle of cosmological proportions is not 180º. Abstract geometries of this nature were developed by Gauss, Bolyai, Lobachevsky and Riemann, others in the early 19th century, amid suspicions that Euclidean geometry may not be the correct description of physical space.

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Mathematics • Stage 5 (5.3 pathway)

Measurement and Geometry
Circle Geometry #

Outcome
A student:
- uses and interprets formal definitions and generalisations when explaining solutions and/or conjectures MA5.3-1WM
- generalises mathematical ideas and techniques to analyse and solve problems efficiently MA5.3-2WM
- uses deductive reasoning in presenting arguments and formal proofs MA5.3-3WM
- applies deductive reasoning to prove circle theorems and to solve related problems MA5.3-17MG

Students:

Prove and apply angle and chord properties of circles (ACMMG272)

- identify and name parts of a circle (centre, radius, diameter, circumference, sector, arc, chord, secant, tangent, segment, semicircle) [L]
- use terminology associated with angles in circles such as subtend, standing on the same arc, angle at the centre, angle at the circumference, angle in a segment [L]
- identify the arc on which an angle at the centre or circumference stands [CCT]
- demonstrate that at any point on a circle there is a unique tangent to the circle, and that this tangent is perpendicular to the radius at the point of contact [CCT]
- prove the following chord properties of circles:
  - chords of equal length in a circle subtend equal angles at the centre and are equidistant from the centre
  - the perpendicular from the centre of a circle to a chord bisects the chord
  - conversely, the line from the centre of a circle to the midpoint of a chord is perpendicular to the chord
  - the perpendicular bisector of a chord of a circle passes through the centre
  - given any three non-collinear points, the point of intersection of the perpendicular bisectors of any two sides of the triangle formed by the three points is the centre of the circle through all three points
  - when two circles intersect, the line joining their centres bisects their common chord at right angles [CCT]
  - ‣ use dynamic geometry software to investigate chord properties of circles (Problem Solving, Reasoning) [ICT, CCT]
- prove the following angle properties involving circles:
  - the angle at the centre of a circle is twice the angle at the circumference standing on the same arc
  - the angle in a semicircle is a right angle
  - angles at the circumference, standing on the same arc, are equal
  - the opposite angles of cyclic quadrilaterals are supplementary
  - an exterior angle at a vertex of a cyclic quadrilateral is equal to the interior opposite angle [CCT]
  - ‣ use dynamic geometry software to investigate angle properties of circles (Communicating, Problem Solving, Reasoning) [ICT, CCT]
  - ‣ apply circle theorems to prove that the angle in a semicircle is a right angle (Communicating, Problem Solving, Reasoning) [CCT]

Mathematics • Stage 5 (5.3 pathway)

Measurement and Geometry

Circle Geometry #

- prove the following tangent and secant properties involving circles:
  - the two tangents drawn to a circle from an external point are equal in length
  - the angle between a tangent and a chord drawn to the point of contact is equal to the angle in the alternate segment
  - when two circles touch, their centres and the point of contact are collinear
  - the products of the intercepts of two intersecting chords of a circle are equal
  - the products of the intercepts of two intersecting secants to a circle from an external point are equal
  - the square of a tangent to a circle from an external point equals the product of the intercepts of any secants from the point [CCT]

  ‣ use dynamic geometry software to investigate tangent properties of circles (Communicating, Problem Solving, Reasoning) [ICT, CCT]

- apply circle theorems to find unknown angles and sides in diagrams [N, CCT]

Background information

As well as solving arithmetic and algebraic problems in circle geometry, students should be able to reason deductively within more theoretical arguments. Diagrams would normally be given to students, with the important information labelled on the diagram to aid reasoning. Students would sometimes need to produce a clear diagram from a set of instructions.

Attention should be given to the logical sequence of theorems and to the types of arguments used. Memorisation of proofs is not intended. Ideally, every theorem presented should be preceded by a straight-edge-and-compasses construction to confirm the theorem, before it is proven in a manner appropriate to the student's work level by way of an exercise or an investigation.

The tangent-and-radius theorem is difficult to justify at this stage and is probably better taken as an assumption as indicated above.

Circle Geometry may be extended to examining the converse of some of the theorems related to cyclic quadrilaterals, leading to a series of conditions for points to be concyclic. However, students may find these results difficult to prove and apply.

The angle in a semicircle theorem is also called Thales' theorem because it was traditionally ascribed to Thales (c624–548 BC) by the ancient Greeks, who reported that it was the first theorem ever proven in mathematics.

Mathematics • Stage 5 (5.3 pathway)

Statistics and Probability
Single Variable Data Analysis

Outcome
A student:
- uses and interprets formal definitions and generalisations when explaining solutions and/or conjectures MA5.3-1WM
- generalises mathematical ideas and techniques to analyse and solve problems efficiently MA5.3-2WM
- uses deductive reasoning in presenting arguments and formal proofs MA5.3-3WM
- uses standard deviation to analyse data MA5.3-18SP

Students:

Calculate and interpret the mean and standard deviation of data and use these to compare data sets (ACMSP278)

- investigate the meaning and calculation of standard deviation using a small set of scores [L, N, ICT]
  - explain why the standard deviation is calculated using the squares of all $\left(x-\bar{x}\right)$ (Communicating, Reasoning) [N]
- find the standard deviation of a set of scores using digital technologies [L, ICT]
  - investigate and describe the effect on the standard deviation, if any, of adding a score to the data set, such as adding a score equivalent to the mean, or adding a score more/less than one standard deviation from the mean (Problem Solving, Reasoning) [N, ICT, CCT]
  - investigate and describe the effect on the standard deviation, if any, of altering all the scores in the data set by operations such as doubling all scores or adding a constant to all scores (Problem Solving, Reasoning) [N, ICT, CCT]
- use the mean and standard deviation to compare two sets of data [N, CCT]
  - compare the spread of data sets with the same mean but different standard deviations (Communicating, Reasoning) [N, CCT]
  - describe the relative spread of data sets with different means by referring to standard deviation (Communicating, Reasoning) [N, CCT]
- compare the relative merits of the range, interquartile range and standard deviation as measures of spread [N, CCT]
  - determine which measure of spread is most appropriate when the set of scores includes an outlier (Communicating, Reasoning) [CCT]

Background information

It is intended that students develop a feeling for the concept of standard deviation being a measure of spread of a symmetrical distribution without going into detailed analysis. When using a calculator the $\boxed{\sigma_n}$ button for standard deviation of a population will suffice.

Mathematics • Stage 5 (5.3 pathway)

Statistics and Probability
Bivariate Data Analysis

Outcome
A student:
- uses and interprets formal definitions and generalisations when explaining solutions and/or conjectures MA5.3-1WM
- generalises mathematical ideas and techniques to analyse and solve problems efficiently MA5.3-2WM
- investigates the relationship between numerical variables using lines of best fit, and explores how data is used to inform decision making processes MA5.3-19SP

Students:

Use information technologies to investigate bivariate numerical data sets. Where appropriate use a straight line to describe the relationship allowing for variation (ACMSP279)

- use ICT to construct a line of best fit for bivariate numerical data in a spreadsheet [N, ICT]
  - investigate different methods of constructing the line of best fit using ICT (Problem Solving) [ICT, CCT]
- use lines of best fit to estimate what might happen between known data values (interpolation) and predict what might happen beyond known data values (extrapolation) [N, ICT, CCT]
  - compare predictions obtained from different lines of best fit (Problem Solving) [N, ICT, CCT]

Investigate reports of studies in digital media and elsewhere for information on their planning and implementation (ACMSP277)

- investigate and evaluate the appropriateness of sampling methods and sample size in reports where statements about population are based on a sample [N, CCT]
  - determine whether the sample used enables inferences or conclusions to be drawn about the population in general (Reasoning) [CCT]
- critically review surveys, polls and media reports [N, CCT]
  - identify, describe and evaluate issues such as the misrepresentation of data, apparent bias in reporting or sampling techniques, or issues with the questions posed to collect the data (Communicating, Problem Solving, Reasoning) [N, CCT, EU]
  - discuss issues to be considered in the implementation of policies that result from studies reported in the media or elsewhere (Communicating, Problem Solving) [N, CCT, EU]
- investigate the use of statistics and associated probabilities in shaping decisions made by governments and companies, eg setting of insurance premiums or the use of demographic data to determine where and when various facilities may be built [N, CCT]
  - use Australian census data to identify issues for the local area or state and suggest implications for future planning in the local area or state (Problem Solving, Reasoning) [CCT, EU]