PATTERNS

1. Complete these sentences with even or odd.

a) Adding 1 to an even number gives an ______ number

b) An even number multiplied by 1 gives an ______ number

c) The sum of 2 odd numbers gives an ______ number.

d) The product of an odd number and 2 is an ______ number

e) The product of an even number and 2 is an ______ number

f) If 0 is added to an even number the answer is ______

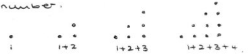
g) If 1 is subtracted from an even number the answer is ______

h) Between 6 and 8 there is only one ______ number.

i) The sum of an odd number and an even number is ______

j) If an even number is multiplied by an odd number the answer is an ______ number.

2 Triangular Numbers



a) What is the next triangular number? Draw a diagram to represent it.

b) Write the 8th triangular number

c) What numbers would you add to get the 10th triangular number

3. Square Numbers

a) Draw a diagram to represent the 7th square number.

b) Describe, in your own words, how each term is formed.

c) Copy this pattern into your book. Extend it 6 more lines

1 = 1

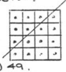
1 + 3 = 4

1 + 3 + 5 = 9

d) What square number would you get if you added the first 20 odd numbers?

4. The first number which is both triangular and square is 1
What is the next number which is both?

5 Square numbers can be written as the sum of 2 triangular numbers
Draw a diagram to show how each of these square numbers is the sum of two triangular numbers: a) 4 b) 9 c) 49.



16 = 6 + 10

6 Perfect Numbers

6 = 1 + 2 + 3   6 = 1 × 2 × 3

Because the sum of the divisors of 6 (except for 6 itself) is 6, it is called a perfect number. The smallest perfect number is 6. Work out the second. (Hint: It lies between 20 and 30)

4.1 Pascal's triangle

This triangle of numbers was named after the French mathematician Blaise Pascal (1623–62). Some interesting patterns emerge when using Pascal's triangle. Can you see how each new row is formed?

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
... ...
... ...
... ...
... ...

Part A

1 Complete the next four rows of Pascal's triangle.

2 Complete the table below for the sums of each row in Pascal's triangle.

Row	1	2	3	4	5	6	7	8
Sum	1	2	4	8				

3 Predict the sum of the numbers for each of the next five rows of the triangle. _____ _____ _____ _____ _____

4 What do you think is the sum of the 20th row? _____

5 **a** Add *all* the numbers in the first 3 rows of the triangle _____
b Add *all* the numbers in the first 4 rows of the triangle _____
c Add *all* the numbers in the first 5 rows of the triangle _____
What do you notice? _______________________________

6 What do you think is the total of all the numbers in the first:
a 12 rows of the triangle? _____
b 20 rows of the triangle? _____

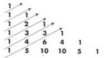
7 Using a coloured pen, circle all the even numbers in Pascal's triangle. A nice geometric pattern of triangles should occur.

Part B

Within Pascal's triangle we can also find other number patterns. Can you see the triangular numbers 1, 3, 6, 10, 15, ...?

What about the Fibonacci numbers 1, 1, 2, 3, 5, 8, ...?

To help you find the Fibonacci numbers we need to rewrite the triangle above as a right-angled triangle. Add along the arrows to obtain the Fibonacci numbers.



4.2 Which number am I?

1 I am a square number. When one is added to me the answer is a prime number. Which square number am I?
Note: There is more than one answer. ______________

2 I am a triangular number. If you add one to me the answer is a square number. Which triangular number am I? ________

3 There are three square numbers which, when added to a particular cube number, give triangular numbers.
Find the three square numbers. ______________

4 We are two prime numbers. If you add us you will obtain 12. Which numbers are we? ________

5 We are two prime numbers which add to give 45. Which numbers are we? ________

6 Two prime numbers when multiplied give an answer which ends in a zero. Find the two numbers. ________

7 Belinda multiplied two prime numbers. The answer ended in 2. Which numbers did she use? ________

8 Frank multiplied two prime numbers. The answer ends in 3, but neither of the numbers ends in 3.
What could the numbers be? ________

9 Maths to the rescue

Anna was walking to her grandmother's home carrying a basket of eggs. Prince Charming, who was riding a horse, hit the basket and broke all the eggs. Wishing to pay for the damage, he asked her how many eggs she had. Anna did not know, but remembered that, when she counted them by twos, there was one egg left over. When she counted them by threes, there was also one left over. When she counted them by fours, there was again one left over. But when she counted them by fives, there were no eggs left over.

How many eggs did Anna have if you know that the basket could carry no more than four or five dozen eggs? ________

Patterns with numbers

Multiples

You will need a number board similar to the one below, and counters or small discs to cover the numbers.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1 On your number board cover all the multiples of 4. You will find that these numbers form a regular pattern.

2 Discover if the multiples of the numbers up to 12 also form patterns.

Use your number boards as posters to decorate your classroom.

Square numbers

			7	6	5				
			8	1	4				
			9	2	3	14			
			10	11	12	13			

1 Discover how this spiral number square has been started.

2 Complete this pattern until you reach the number 100.

3 Shade all the squares which show square numbers.

You should be able to see a pattern.

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Why are policemen strong?

Find the next number in the sequences below, and exchange it for the letter starting each sequence to decode the puzzle answer.

- **(A)** 2, 5, 11, 23, ☐
- **(B)** 2, 4, 16, ☐
- **(C)** 7, 13, 19, ☐
- **(D)** 19, 16, 13, ☐
- **(E)** 4, 8, 20, 56, ☐
- **(F)** 2, 2, 4, 6, 10, 16, ☐
- **(H)** 1, 1, 2, 4, 7, 13, ☐
- **(I)** 3, 6, 12, 24, ☐
- **(L)** 10, 11, 9, 12, 8, ☐
- **(N)** 2, 6, 18, 54, ☐
- **(O)** 20, 19, 17, ☐
- **(P)** 2, 3, 5, 7, 11, 13, ☐
- **(R)** 13, 26, 39, ☐
- **(S)** 5, 7, 13, 31, ☐
- **(T)** 1, 1, 2, 4, 7, 13, 24, ☐
- **(U)** 1, 1, 1, 2, 3, 4, 6, 9, 13, ☐
- **(Y)** 1, 2, 2, 4, 3, 6, 4, 8, 5, 10, ☐

256	164	25	47	19	85	164	44	24	164	6	25	47	162

24	14	13	10	19	17	44	52	47	26	26	48	25

Why do elephants have big ears?

24

Answer the questions below to find the code which you can use to answer the puzzle given above.

A 2^3 = ____________ O $\sqrt{36}$ = ____________

A 3^2 = ____________ O $\sqrt[3]{343}$ = ____________

D 5^3 = ____________ O $\sqrt{121}$ = ____________

D 10^2 = ____________ P $\sqrt{144}$ = ____________

D $\sqrt{100}$ = ____________ R 13^2 = ____________

E $\sqrt[3]{125}$ = ____________ S $\sqrt{400}$ = ____________

H $\sqrt[3]{8}$ = ____________ T 10^3 = ____________

L $\sqrt{16}$ = ____________ T $2^2 \times 3^2$ = ____________

M $\sqrt[3]{27}$ = ____________ U $5^2 \times 2^4$ = ____________

N 4^3 = ____________ W $\sqrt{900}$ = ____________

N 9^2 = ____________ Y $\sqrt{1}$ = ____________

N 2^5 = ____________ Y 3^3 = ____________

64	6	125	10	1	30	7	400	4	100	’ 81	1000
12	8	27	36	2	5	169	9	32	20	11	3

2.1 Odd and even numbers

A **remainder** is the amount left over in a division.

An **even number** has no remainder when it is divided by 2.
We say an even number is divisible by 2.

An **odd number** has a remainder of 1 when it is divided by 2.
An odd number is ***not*** divisible by 2.

1 The first four **even numbers** are **2**, **4**, **6** and **8**. Write the *next* six even numbers.

______ ______ ______ ______ ______ ______

2 The first four **odd numbers** are **1**, **3**, **5** and **7**. Write the *next* six odd numbers.

______ ______ ______ ______ ______ ______

3 Circle the **even numbers**.

26 36 21 44 19 38 30 43 37 42

4 Circle the **odd numbers**.

17 22 35 40 21 26 29 37 48 43

5 Look at your answers to questions 1 to 4 and complete these statements.

a An **even number** must have a final digit which is ___, ___, ___, ___ or ___.

b An **odd number** must have a final digit which is ___, ___, ___, ___ or ___.

6 Are the following numbers **even** or **odd**?

a 15 __________ **b** 24 __________ **c** 86 __________

d 93 __________ **e** 106 __________ **f** 159 __________

g 332 __________ **h** 256 __________ **i** 953 __________

j 1 235 __________ **k** 4 636 __________ **l** 7 398 __________

7 Write:

a the next even number after 22. ____

b the next odd number after 15. ____

c the next odd number after 29. ____

d the next even number after 38. ____

8 Write:

a the next three **even** numbers after **14**. ____ ____ ____

b the next three **odd** numbers after **19**. ____ ____ ____

2.1 Odd and even numbers

CONTINUED

9 Complete these additions.

a $6 + 8 =$ ____ **b** $8 + 10 =$ ____ **c** $14 + 6 =$ ____ **d** $16 + 8 =$ ____

e $7 + 3 =$ ____ **f** $11 + 9 =$ ____ **g** $13 + 17 =$ ____ **h** $19 + 15 =$ ____

10 Use the words *even* or *odd* to complete these statements.

a An even number plus an even number makes an ________ number.

b An odd number plus an odd number makes an ________ number.

11 Complete these additions.

a $6 + 5 =$ ____ **b** $8 + 7 =$ ____ **c** $12 + 9 =$ ____ **d** $16 + 11 =$ ____

e $5 + 8 =$ ____ **f** $9 + 6 =$ ____ **g** $15 + 8 =$ ____ **h** $19 + 16 =$ ____

12 Use the words *even* or *odd* to complete these statements.

a An even number plus an odd number makes an ________ number.

b An odd number plus an even number makes an ________ number.

13 Complete these multiplications.

a $4 \times 6 =$ ____ **b** $8 \times 2 =$ ____ **c** $6 \times 10 =$ ____ **d** $12 \times 8 =$ ____

e $7 \times 3 =$ ____ **f** $9 \times 5 =$ ____ **g** $11 \times 7 =$ ____ **h** $15 \times 9 =$ ____

14 Use the words *even* or *odd* to complete these statements.

a An even number times an even number makes an ________ number.

b An odd number times an odd number makes an ________ number.

15 Complete these multiplications.

a $6 \times 3 =$ ____ **b** $8 \times 5 =$ ____ **c** $10 \times 9 =$ ____ **d** $12 \times 7 =$ ____

e $5 \times 4 =$ ____ **f** $7 \times 6 =$ ____ **g** $11 \times 8 =$ ____ **h** $15 \times 6 =$ ____

16 Use the words *even* or *odd* to complete these statements.

a An even number times an odd number makes an ________ number.

b An odd number times an even number makes an ________ number.

Fractal Automata

Materials: special "brick-grid" mat (following each problem page), graph paper, colored pencils, markers, or crayons (alternatively, *lots* of colored cubes or bricks)

Math: discrete math, formal systems, recursion, cellular automata, fractals

This cluster is less about solving a problem than about following directions carefully and producing a group art project. We don't expect middle grade students to analyze cellular automata or fractals in earnest (or even to use those terms) but we can give them experiences that will help them understand the concepts—and to feel so comfortable with the mathematical guts that they don't get blown away by the vocabulary.

Each group member has one or more "coloring rules" to follow. All each person has to do is look for a pattern of bricks, and color the brick above it if they find it. As more bricks get colored in, more of their pattern appears, and gradually the big pattern emerges. In that way, this activity has some of the same satisfaction of knitting (if you like it) where a period of careful work without deep thought produces something ordered and wonderful.

The trick is to avoid coloring in the bricks you aren't responsible for. Consider asking students to check each other.

These are fragile activities in the sense that it's easy to make a single mistake that changes the eventual result. But as long as no one makes *consistent* mistakes, the sorts of patterns that emerge should be like those on a "correct" page. This is a wonderful property of chaotic systems. The flap of a butterfly's wings in China may indeed change the weather in New York, but the clouds that appear will inevitably look like clouds.

Some Questions

To follow up on the mathematics, *you* will have to ask questions of the groups. Here are some suggestions:

- What patterns do you see?
- How does your design compare with other groups'? What's the same? What's different?
- With this many colors (and this many bricks controlling the outcomes) how many rules are there altogether? How do you know?
- What would it have looked like if you put a red brick *here* in the bottom row?
- What would it have looked like if the bottom row started out all blue?
- Were there any rules you never used?
- What effect do the edge rules have?

Sierpinski Bricks

This is the classic. There are only four rules, and they make the amazing, self-similar Sierpinski Triangle. This is the same pattern you get when you color Pascal's Triangle one color for even and one for odd. So there are extensions: do the exercise for Pascal's Triangle and challenge the class to explain why the patterns are the same.

Another important extension is to let students invent their own coloring rules (start simple: two colors, four rules, one for each possible combination of two bricks) to see that the Sierpinski rules are special: most others make boring—or at least more predictable—patterns.

Brackets!

This has a different sort of pattern, more complex. There are still two colors, but each new block is determined by *three* blocks instead of two. To make rules for all combinations of three blocks means eight rules altogether, spread among the four cards. Since we're using three blocks, the underlying pattern is a grid rather than a staggered "brick" pattern. Use page 45 for the students to color.

We think this pattern repeats after 14 rows if you color the bottom row all yellow. But if you don't—for example, if you add two red blocks at the edge of the bottom row—you get a different effect. Let groups compare their patterns to others with slightly different starting conditions.