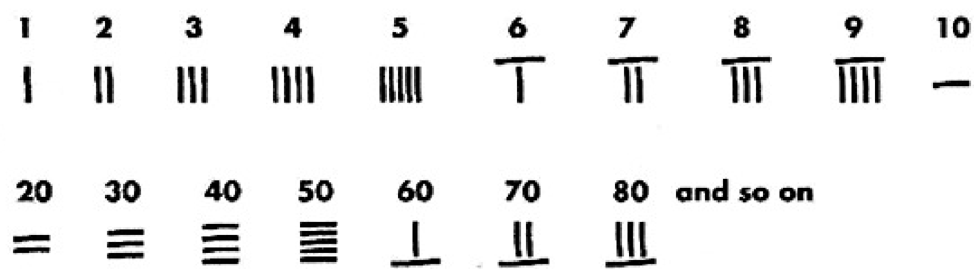
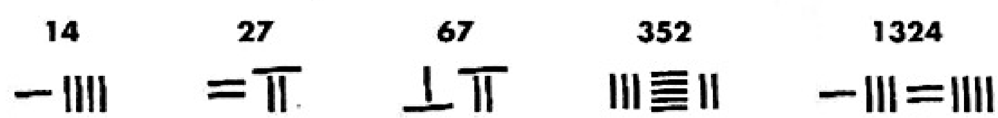
3.1 Ancient Chinese number system

Early numerals must have been very clumsy to use because every object was usually shown by one stroke. The ancient Chinese managed to solve this problem in a very clever way.

Here are their numerals.



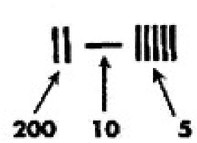
Note: There is no symbol for 100 or 1000. Study the following numerals and discuss how the Chinese overcame this problem.



To solve this difficulty the Chinese used the vertical rods to represent hundreds as well as units. Horizontal rods represented thousands and tens.

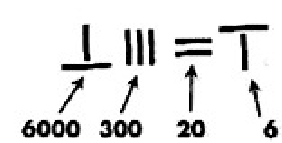
Example 1

215



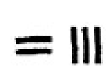
Example 2

6326



1 What do these ancient Chinese rod numerals mean?

a 

b 

c 

d 

e 

f 

3.1 Ancient Chinese number system (continued)

2 Write Chinese rod numerals for the following.

a 6 **b** 13

c 62 **d** 113

e 378 **f** 4275

Note: There is no numeral for zero.

3 Write the answers to these in ancient Chinese rod numerals.

a $7 \times 12 =$ **b** $273 - 127 =$

c $65 \times 27 =$ **d** $975 \div 25 =$

e $1257 + 3704 =$ **f** $14 \times 9 + 302 =$

4 Calculate answers to these. Write them in ancient Chinese rod numerals.

a 𝍤 + 𝍪𝍡 =

b 𝍩𝍢𝍪𝍣 ÷ 𝍣 =

c 𝍡𝍯𝍥 × 𝍢 =

d 𝍧𝍮𝍡 − 𝍡𝍫𝍨 =

e 𝍪𝍧 × 𝍩𝍡 =

f 𝍮𝍡 − 𝍨 ÷ 𝍨 =

3.4 The binary system
(Base 2 arithmetic)

While we have ten fingers, electrical circuits have two states, on or off, so they use a base 2 or binary system which has only two digits, 0 and 1. Computers run on electricity, so they all use binary languages.

Equivalent base 2 number for the first eight base 10 numbers are:

Base 10	1	2	3	4	5	6	7	8
Base 2	1	10	11	100	101	110	111	1000

Example 1

To change 100110_2 to a base 10 number:

2^5	2^4	2^3	2^2	2	1
1	0	0	1	1	0

$100110_2 = 1 \times 2^5 + 1 \times 2^2 + 1 \times 2 = 32 + 4 + 2 = 38$ in base 10

Example 2

To change 57 in base 10 to base 2:

```
2 ) 57
2 ) 28 | 1
2 ) 14 | 0
 2 ) 7 | 0
 2 ) 3 | 1
 2 ) 1 | 1 ↑
       | 1 |
```

Note: the digits in the answer are written from the bottom upwards.

So 57_{10} is 111001_2.

1 Change these base 2 numbers into base 10 numbers.

a 1010 **b** 1110 **c** 1110011

d 100111 **e** 111111

2 Change these base 10 numbers into base 2 numbers.

a 15 **b** 25 **c** 79

d 150 **e** 1000

3 Complete these tables for addition and multiplication in base 2.

+	0	1
0		
1		

×	0	1
0		
1		

4 Do these base 2 sums and write your answers in base 2.

a 111 + 101 **b** 1011 – 101

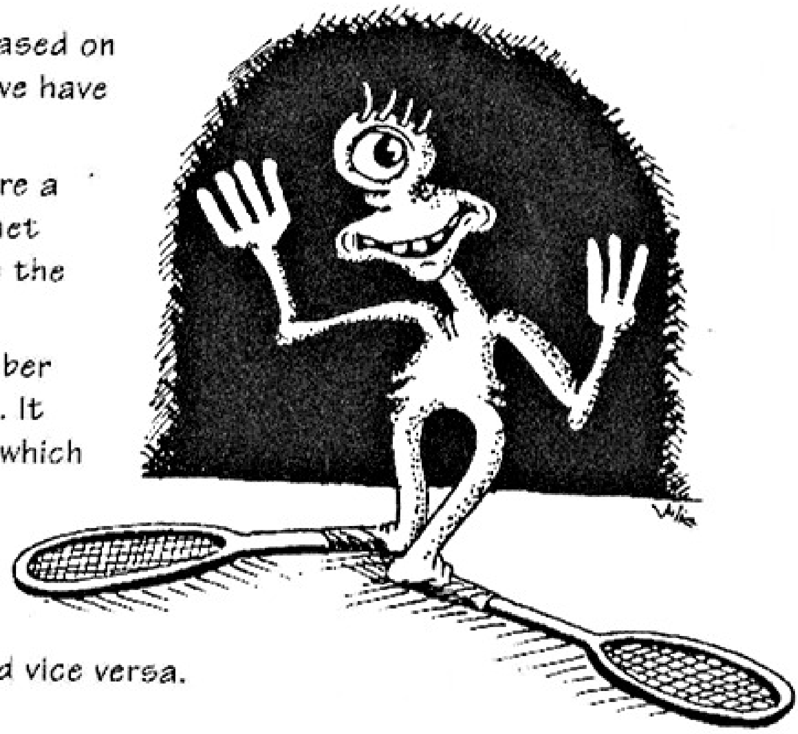
c 101 × 10 **d** 1111 × 11

3.3 Base 7 arithmetic

Our number system is based on tens, probably because we have ten fingers.

Imagine, instead, you were a Sagassian from the planet Sagassia and looked like the creature in the figure.

You might develop a number system based on sevens. It would have seven digits, which when translated into 'Earthspeak', would be 0, 1, 2, 3, 4, 5 and 6. These numbers can be changed into base 10 and vice versa.



Example 1

The number 4065 in base 10 is:

Thousands 10^3	Hundreds 10^2	Tens 10	Units 1
4	0	6	5

$4065_{10} = 4 \times 10^3 + 0 \times 10^2 + 6 \times 10 + 5 \times 1$

The number 4065 in base 7 is:

7^3	7^2	7	1
4	0	6	5

$$\begin{aligned} 4065_7 &= 4 \times 7^3 + 0 \times 7^2 + 6 \times 7 + 5 \times 1 \\ &= 1372 + 0 + 42 + 5 \\ &= 1419_{10} \end{aligned}$$

So 4065_7 is 1419 in base 10.

Example 2

To change 4065_{10} into base 7

```
7)4065
 7)580 | 5
   7)82 | 6
   7)11 | 5
      1 | 4  ↑
      0 | 1
```

Note: the digits in the answer are written from the bottom upwards.

So 4065_{10} is 14565_7.

3.3 Base 7 arithmetic (continued)

1 Change these Sagassian (base 7) numbers into base 10 numbers.

a 23_7 **b** 60_7 **c** 125_7

d 426_7 **e** 5134_7 **f** 3001_7

2 Change these base 10 numbers into base 7 numbers.

a 22 **b** 83 **c** 204

d 359 **e** 1227 **e** 2000

3 Complete these tables for addition and multiplication in base 7.

+	0	1	2	3	4	5	6
0							
1							
2							
3							
4							
5							
6							

×	0	1	2	3	4	5	6
0							
1							
2							
3							
4							
5							
6							

4 Do these sums in base 10 and give the answer in base 7.

a
$$\begin{array}{r} 43 \\ +52 \\ \hline \end{array}$$

b $4 \times 3 =$

c
$$\begin{array}{r} 64 \\ -35 \\ \hline \end{array}$$

d
$$\begin{array}{r} 22 \\ \times \ \ 5 \\ \hline \end{array}$$

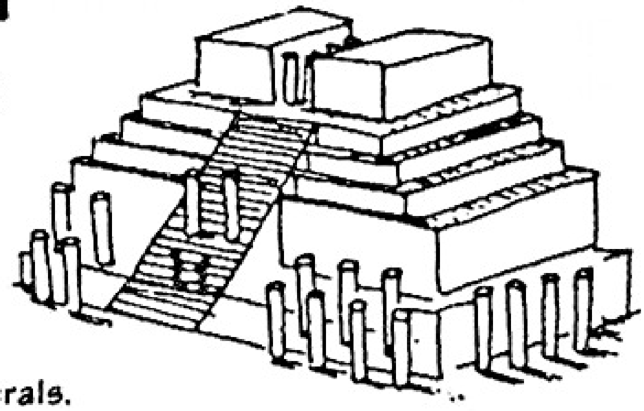
e
$$\begin{array}{r} 261 \\ 104 \\ +\ \ 32 \\ \hline \end{array}$$

f
$$\begin{array}{r} 410 \\ \times \ \ 6 \\ \hline \end{array}$$

5 What digits would be used for:

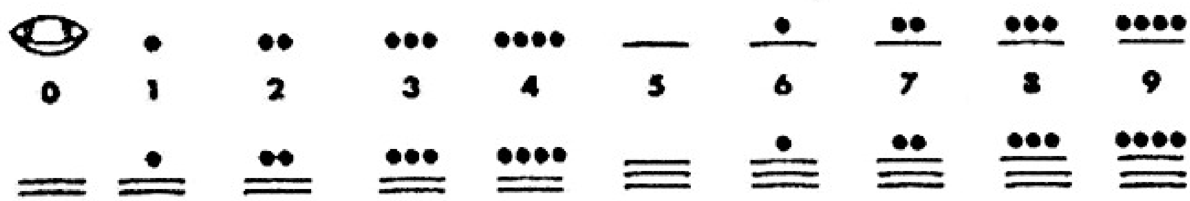
a base 5? **b** base 4? **c** base 6?

3.2 Mayan number system



Other number systems we have looked at began in Europe or Asia, but the Mayans lived in Central America about 2400 years ago. They invented a system which used bars and also had a symbol for zero.

Here are the Mayan bar and dot numerals.



- For numbers over 19, the Mayans wrote their symbols in groups vertically, from the top down.
- If the number has three symbols then:
  - the top symbol is multiplied by 360
  - the next symbol down is multiplied by 20
  - the next symbol down is multiplied by 1
- If the number has two symbols then:
  - the top symbol is multiplied by 20
  - the next symbol down is multiplied by 1

Example 1

means

$1 \times 360 = 360$

$5 \times 20 = 100$

$11 \times 1 = 11$

471

Example 2

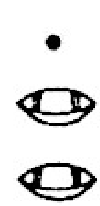
means

$9 \times 20 = 180$

$0 \times 1 = 0$

180

Example 3



means

$1 \times 360 = 360$

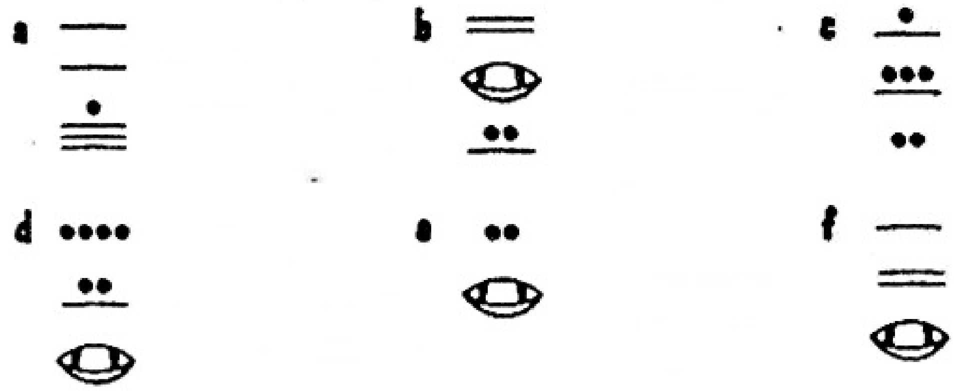
$0 \times 20 = 0$

$0 \times 1 = 0$

360

1 Write the following Mayan numbers in our system.

a b c d e f



Note: As you can see the position of the numeral makes a lot of difference to its value!

2 Write each of these numbers using the Mayan system.

a 25 b 109 c 384

d 1000 e 484 f 925

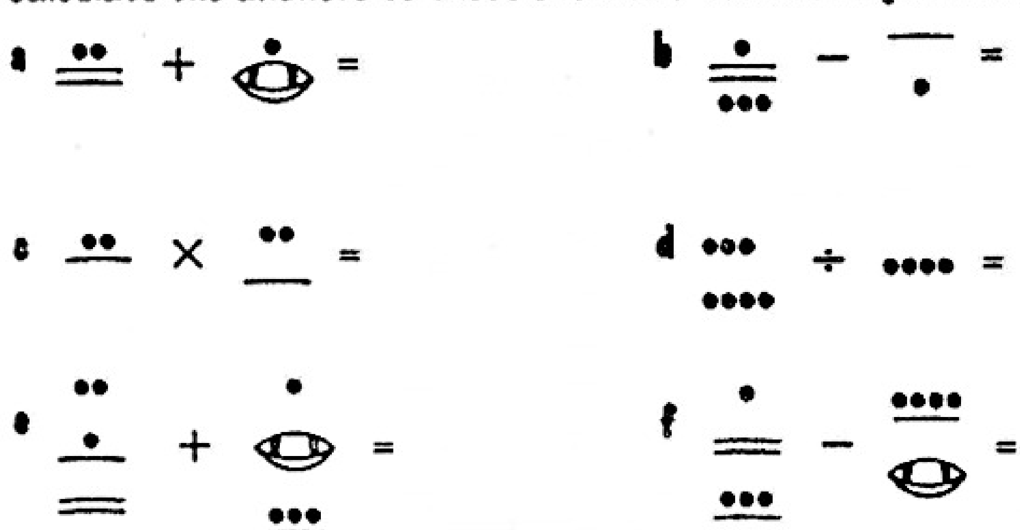
3 Write the answers to these using Mayan numbers.

a $6 \times 9 =$ b $195 + 516 =$ c $15 \times 39 =$

d $876 \div 4 =$ e $622 - 587 =$ f $195 - (4 \times 8) =$

4 Calculate the answers to these and write them in Mayan numerals.

a + = b − = c × = d ÷ = e + = f − =



NUMBER SYSTEMS

1 Roman addition

Name:

1 Complete this addition chart for Roman numerals:

+	I	II	III	IV	V	VI	VII	VIII	IX	X
I	II									
II					VII					
III								XI		
IV										
V										
VI										
VII			X							
VIII								XVI		
IX										
X										XX

Suggest why the Romans used 'V' for *five* and 'X' for *ten*. (Use a reference book to help you check your ideas.)

....

....

....

Use your addition chart to find:

a IX + V = ____ b VIII + V = ____ c IV + IV = ____ d VI + VI = ____ e III + VII = ____

Find these:

a XXIII + IV = ____ b XXIX + VI = ____ c XIX + XXV = ____

5 Solve these *subtraction* examples:

a XVI − VIII = ____ b XIII − VI = ____

Does the Roman number system:

a group in tens? b group in any other amount? c have place value?

Why isn't there a zero in the addition chart at the top of this page?

2 Egyptian counting

Over 5000 years ago, people in Egypt used these symbols to write numbers:

Numbers 1 to 9	*Ten*	*Hundred*	*Thousand*
a	**b**	**c**	**d**
𓏺 = 1	𓎆 = 10	𓍢 = 100	𓆼 = 1000
𓏺𓏺𓏺𓏺 = 4	⇩	⇩	⇩
𓏺𓏺𓏺𓏺𓏺𓏺𓏺 = 7	𓏺𓏺𓏺𓏺𓏺𓏺𓏺𓏺𓏺𓏺 = 𓎆	𓎆𓎆𓎆𓎆𓎆𓎆𓎆𓎆𓎆𓎆 = 𓍢	𓍢𓍢𓍢𓍢𓍢𓍢𓍢𓍢𓍢𓍢 = 𓆼

1 Write decimal numerals for:

a 𓎆𓎆𓏺𓏺𓏺 = ... b 𓎆𓎆𓎆𓏺𓏺 = ... c 𓎆𓎆𓎆𓎆𓎆𓎆 = ...

d 𓍢𓎆𓎆𓎆𓎆𓎆𓏺𓏺𓏺𓏺 = ... e 𓍢𓍢𓍢𓍢𓎆𓎆 = ... f 𓍢𓍢𓏺𓏺𓏺𓏺𓏺 = ...

g 𓆼𓍢𓍢𓎆𓎆𓎆𓎆𓏺𓏺𓏺 = ... h 𓆼𓆼𓏺𓏺𓏺 = ... i 𓆼𓆼𓆼 = ...

2 Write Egyptian numerals for:

a 40 = ... b 12 = ... c 34 = ...

d 123 = ... e 209 = ... f 4005 = ...

3 Find the answers, in Egyptian numerals, for the following examples:

a 𓎆 + 𓎆𓎆

b 𓎆𓎆𓎆𓏺𓏺𓏺 + 𓎆𓎆𓏺𓏺

c 𓍢𓎆𓏺𓏺𓏺 + 𓍢𓍢𓎆𓎆𓏺

d 𓍢𓎆𓎆𓏺𓏺𓏺𓏺𓏺 + 𓎆𓎆𓎆𓏺𓏺𓏺𓏺𓏺

e 𓎆𓎆𓎆𓏺𓏺𓏺𓏺𓏺 − 𓎆𓏺𓏺

f 𓎆𓎆𓎆 − 𓏺𓏺𓏺

g 𓍢𓎆𓎆 − 𓍢𓏺𓏺𓏺𓏺

h 𓍢𓏺𓏺 − 𓎆𓎆𓎆𓎆

4 Use a reference book to find out why the Egyptians used the symbols 𓎆, 𓍢, and 𓆼.

5 Does the Egyptian number system:

a group in tens? .... b group in any other value? .... c have place value? ....

6 Did the Egyptians have a symbol for zero? (Explain.)

....

....

....

....

Revision.

1. Write in Hindu-Arabic numbers.

a) MC   b) XIV   c) $\overline{C}$ d) MDCCXLIV

2. Write in Roman numerals

a) 27   b) 75   c) 142   d) 2004

3. Write the numeral for

a) six hundred and twenty seven

b) two thousand and forty four

c) two million, three hundred and two thousand

4. Write in simplest form

a) $6 \times 10^3 + 2 \times 10^2 + 3 \times 10 + 4 \times 1$

b) $5 \times 10^3 + 3 \times 10^2 + 5 \times 1$

c) $7 \times 10^4 + 5 \times 10^2$

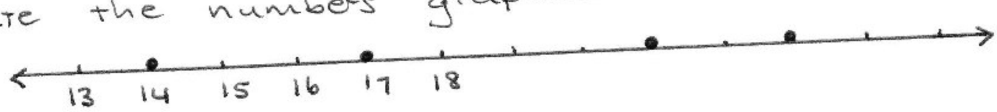
5. Write in expanded form using index notation

a) 408

b) 9324

c) 5015

6. State the numbers graphed on this number line



7. Graph the following numbers on the number line below
{0, 1, $1\frac{1}{2}$, 3}

8. What do the following symbols mean

a) $<$

b) $\neq$

c) $\therefore$

d) $\approx$

9. Insert < = or > to make each statement true

a) 18 ☐ 3×6

b) 2×6 ☐ 13

c) $12 - 3$ ☐ 3^2

d) 3×7 ☐ 4×5

10. Arrange the following numbers in ascending order

267, 285, 257, 305, 212

11 a) Find the product of 8 and 2

b) quotient of 8 and 2

c) sum of 8 and 2

d) Increase 18 by 9

e) What is the difference between 20 and 4

12. Simplify

a) $8 + 2 \times 6$

b) $(15 - 8) \times 2$

c) $12 \div 4 \times 3$

d) $12 - 3 + 9$

e) $4 \times 17 \times 25$

f) $18 \times 9 + 18 \times 91$

13. Insert grouping symbols to make the statement true.

a) $8 + 4 - 3 \times 2 = 6$

b) $8 + 4 - 3 \times 2 = 18$

14. What is the remainder when 37 is divided by 5?

15. Divide 117 by 4.

16 Write a number 100 more than 927.

17. Increase 84 by the product of 8 and 12.

18 Write in our numbers

a) 32_4

b) 10111_2

c) 421_5

19. Write in base 2.

a) 8

b) 17

c) 65

20. Write in base 5

a) 7

b) 30

c) 147