1 .The Prime Race

Materials Required

- A 'race track' using masking tape at approximately one pace length intervals on the ground. Ten to fifteen intervals are sufficient.
- Four willing students.

Rationale

The prime race provides a rich investigation of the generation and distribution of prime numbers. The activity is accessible at a number of different levels. It contributes to the development of mental arithmetic strategies as students try to determine whether each number called out is prime or composite. It provides a highly visual demonstration that 2 is the only even prime. It assists in the recognition and algebraic representation of sequences of numbers. By showing algebraically that some number patterns can never produce primes, it provides an introduction to, and application of, algebraic factorisation.

The physical involvement in the initial race is a powerful means of enhancing students' understanding of prime numbers.

The enrichment activities help students to develop a sense of the rich history of mathematics, and can lead to ideas of proof.

The Activity

Select four voluteer 'race-horses', and line them up at the start of the race. The 'race-horses' commence counting, in turn from 1. Each time a 'horse' says a prime number he or she may move forward one pace.

Mathematical Ideas Developed Through This Activity

- Factors
- Prime numbers
- Composite numbers
- Mersenne primes
- Fermat primes

Student Outcomes

- Development of mental arithmetic patterns and skills
- Recognition that 2 is the only even prime
- Recognition that primes are not evenly distributed
- Use of algebraic notation to describe a number pattern
- Introduction to algebraic factorisation

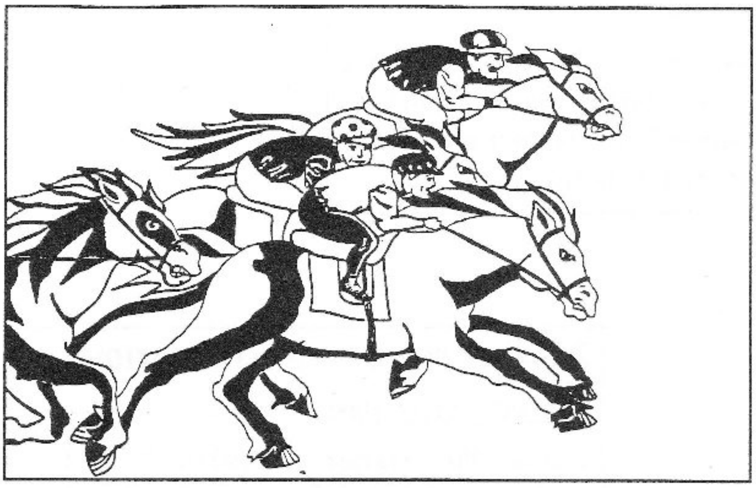
Background Notes For Teachers

An algebraic representation of the pattern of numbers called out by each student is the basis of a mathematical explanation of the results. The four sets of numbers can be represented as $4n - 3$, $4n - 2$, $4n - 1$ and $4n$, or as $4n + 1$, $4n+2$, $4n+3$ and $4n+4$. Since $4n+2=2(n+1)$ and $4n+4=4(n+1)$, these two expressions can never be prime (unless $n = 0$, when $4n + 2$ is prime).

It is not at all clear which of the horses will win in the long run. It turns out that $4n+3$ stays ahead until 26 861, when $4n+1$ moves ahead for just one term. At 26 863, $4n+3$ draws level. The next prime is 26 879, which is of the form $4n+3$, so $4n+3$ takes the lead again. It then leads all the way to 616 841, when $4n+1$ moves back into the lead. The lead changes frequently during the next 17 000 integers, until $4n+3$ again establishes a sound lead. $4n+1$ leads again near twelve and a half million, 1 billion, 6 billion and 18 billion. In the last region it leads for 568 million consecutive terms, leading by 2719 at around 18.7 billion. However $4n+3$ moves back in front at 19 033 524 538. It was established in 1914 that there must be an infinite number of regions in which $4n+1$ leads, but it is not known which leads for the greater proportion of the time. (Lines 1986).

A graphing calculator program for the TI-92 calculator, which gives a real-time visual display of the race, is provided in the References and Resources section, and can be downloaded from the AMTEP Web page at http://www.amt.canberra.edu.au/~sjt/amtep.htm

Student Worksheet-The Prime Race



The Rules

The four people ('horses ') in the race count, in turn, from one. Each time a prime number is called out, the horse calling the number may move one pace forward. The horse reaching the finishing line first is the winner.

Who Wins?

Write down the first five numbers called out by each of the horses in the race.

Horse A:	Horse B:	Horse C:	Horse D:
1	2	3	4
5			

Describe, in words or symbols, the pattern of numbers called out by each horse.

Explain, in words or symbols, why horses B and D are born losers.

Pathways to Enrichment

1. The 6 horse race

Predict which horses will be born losers in a six-horse race. Justify your prediction. Experiment with different numbers of horses to try to predict the born losers.

2. Other ways to generate primes

Quadratic expressions

Examine the sequence of numbers 5, 7, 11, 17, 25. Notice that it contains four primes before the first composite.

- Write down a similar sequence commencing with 11. How far can you go before obtaining a composite number?
- Predict how far the sequence commencing with 17 will go before a composite is obtained. Justify your prediction.
- Find the longest such sequence commencing with a number less than 100.

Fermat primes

- Examine the sequence of numbers 3, 5, 9, 17, 33, ... Describe the pattern of numbers in this sequence. Which of the numbers are primes?
- Predict the next number in the sequence which will be prime, and test your prediction.
- Research Fermat's conjecture that numbers of the form $F_n = 2^{2^n} + 1$ are prime for all n.

Mersenne primes

- Examine the sequence of numbers 1, 3, 7, 15, 31, ... Describe the pattern of numbers in this sequence. Which of the numbers are primes?
- Predict the next number in the sequence which will be prime, and test your prediction.
- Research Mersenne primes, and their role in the search for very large prime numbers.

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