## Exercise 2.1

1 Complete each table of values using the given rules.

a $y = x + 3$

| $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $y$ | | | | |

b $y = 2x + 5$

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $y$ | | | | |

c $y = 3x - 4$

| $x$ | 5 | 6 | 7 | 8 |
|---|---|---|---|---|
| $y$ | | | | |

d $y = 5x - 7$

| $x$ | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| $y$ | | | | |

2 For each table of values in Q1, compare the differences between the $y$-values and the co-efficient of $x$ in the rule. What do you notice?

3 Use the method of finite differences to find a rule for each table of values.

a

| $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $y$ | 4 | 8 | 12 | 16 |

b

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $y$ | 6 | 7 | 8 | 9 |

c

| $x$ | 4 | 5 | 6 | 7 |
|---|---|---|---|---|
| $y$ | 11 | 13 | 15 | 17 |

d

| $p$ | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| $q$ | 5 | 8 | 11 | 14 |

e

| $p$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $q$ | 9 | 14 | 19 | 24 |

f

| $p$ | 7 | 8 | 9 | 10 |
|---|---|---|---|---|
| $q$ | 47 | 54 | 61 | 68 |

g

| $a$ | 4 | 5 | 6 | 7 |
|---|---|---|---|---|
| $b$ | 17 | 19 | 21 | 23 |

h

| $a$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $b$ | 3 | 7 | 11 | 15 |

i

| $a$ | 3 | 4 | 5 | 6 |
|---|---|---|---|---|
| $b$ | 18 | 24 | 30 | 36 |

j

| $s$ | 5 | 6 | 7 | 8 |
|---|---|---|---|---|
| $t$ | 17 | 22 | 27 | 32 |

k

| $s$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $t$ | 13 | 20 | 27 | 34 |

l

| $s$ | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| $t$ | 19 | 31 | 43 | 55 |

### Consolidation

4

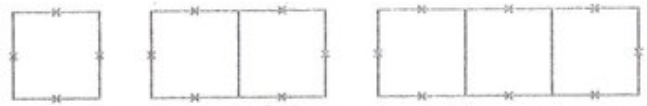
a Copy and complete this table of values.

| Number of pentagons ($p$) | 1 | 2 | 3 |
|---|---|---|---|
| Number of triangles ($t$) | | | |

b Write down an algebraic rule that links the number of triangles ($t$) to the number of pentagons ($p$).

c How many triangles would there be in a figure with 9 pentagons?

5



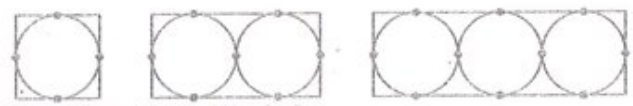
a Copy and complete this table of values.

| Number of squares ($s$) | 1 | 2 | 3 |
|---|---|---|---|
| Number of crosses ($c$) | | | |

b Write down an algebraic rule that links the number of crosses ($c$) to the number of squares ($s$).

c How many crosses would there be in a figure with 20 squares?

6



a Copy and complete this table of values.

| Number of circles ($c$) | 1 | 2 | 3 |
|---|---|---|---|
| Number of dots ($d$) | | | |

b Write down an algebraic rule that links the number of dots ($d$) to the number of circles ($c$).

c How many dots would there be in a figure with 15 circles?

7

a Copy and complete this table of values.

| Number of large rhombuses ($r$) | 1 | 2 | 3 |
|---|---|---|---|
| Number of dots ($d$) | | | |

b Write down an algebraic rule that links the number of dots ($d$) to the number of large rhombuses ($r$).

c How many dots would there be in a figure with 40 large rhombuses?

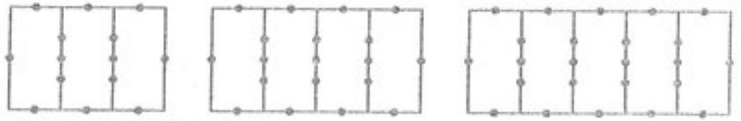
8

a Copy and complete this table of values.

| Number of squares ($s$) | 1 | 2 | 3 |
|---|---|---|---|
| Number of dots ($d$) | | | |

b Complete this rule that relates the number of dots to the number of squares: $d = \Delta s + \square$.

9

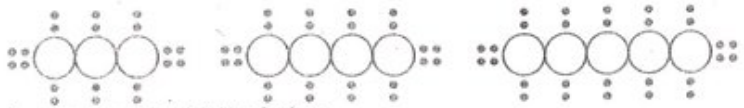


a Copy and complete this table of values.

| Number of rectangles ($r$) | 3 | 4 | 5 |
|---|---|---|---|
| Number of dots ($d$) | | | |

b Complete this rule that relates the number of dots to the number of rectangles: $d = \Delta r + \square$.

10



a Copy and complete this table of values.

| Number of circles ($c$) | 3 | 4 | 5 |
|---|---|---|---|
| Number of dots ($d$) | | | |

b Complete this rule that relates the number of dots to the number of circles: $d = \Delta c + \square$.

11

a Copy and complete this table of values.

| Number of crosses ($c$) | 2 | 3 | 4 |
|---|---|---|---|
| Number of dots ($d$) | | | |

b Complete this rule that relates the number of dots to the number of crosses: $d = \Delta c + \square$.

12 Use the method of finite differences to find a rule linking the $x$- and $y$-values in each table.

a

| $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $y$ | −7 | −14 | −21 | −28 |

b

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $y$ | 5 | 4 | 3 | 2 |

c

| $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $y$ | 7 | 5 | 3 | 1 |

d

| $x$ | 3 | 4 | 5 | 6 |
|---|---|---|---|---|
| $y$ | 11 | 8 | 5 | 2 |

e

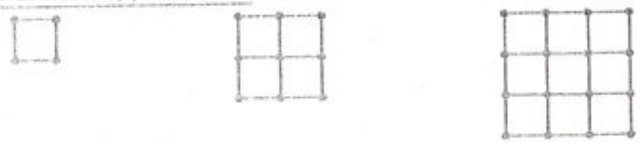
| $x$ | −4 | −3 | −2 | −1 |
|---|---|---|---|---|
| $y$ | 10 | 9 | 8 | 7 |

f

| $x$ | −2 | −1 | 0 | 1 |
|---|---|---|---|---|
| $y$ | 13 | 10 | 7 | 4 |

## Further applications

13



a Copy and complete this table of values.

| Number of squares ($s$) | 1 | 4 | 9 |
|---|---|---|---|
| Number of dots ($d$) | | | |

b Write down an algebraic rule that links the number of dots ($d$) to the number of squares ($s$).

c How many dots would there be in a figure with 64 squares?

14

a Copy and complete this table of values.

| Number of cans in base ($b$) | 1 | 2 | 3 |
|---|---|---|---|
| Total number of cans ($c$) | | | |

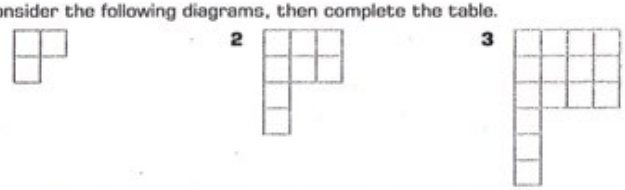
b Write down an algebraic rule that links the total number of cans ($c$) to the number of cans in the base ($b$).

c How many cans would there be in a pile with 10 cans in the base?

**TRY THIS** **Flags**

Consider the following diagrams, then complete the table.

1 2 3



| Pole length | 1 | 2 | 3 | 4 | 5 | ...$n$ |
|---|---|---|---|---|---|---|
| Number of squares | 3 | | | | | |

Find a rule relating the number of squares in the flag to the pole length.

HINT: The rule is not linear.

# Patterns in Science



***Patterns occur everywhere in Science – some people say that Mathematics is the science of patterns. Scientists often look for patterns of behaviour to help them understand what is happening. So perhaps Newton saw the pattern that apples always fell downwards from the tree and so discovered gravity!***

**1** Write the next three numbers in the pattern: 4, 7, 10, ____, ____, ____

**2** Write the next three numbers in the pattern: 9, 5, 1, ____, ____, ____

**3** Write the next three numbers in the pattern: 2, 4, 8, ____, ____, ____

**4** Write the next three numbers in the pattern: 1, 4, 9, ____, ____, ____

**5** Write the next three numbers in the pattern: 1, 3, 6, ____, ____, ____

**6** Beryl has 6 boxes. Each one is in the shape of a cube. Her task is to fill each box with small cubes.

These cubes are all the same size – each edge is 1 cm in length. Beryl has filled 3 boxes so far. The number of small cubes that fit in each box is listed below.

Extend the number pattern by writing the next 3 numbers.

216, 125, 64, ____, ____, ____

**7** Fill in the gaps in the following pattern.

$(1 \times 9) + 2 = 11$

$(12 \times 9) +$ ____ $= 111$

(_____ $\times 9) + 4 = 1111$

(_____ $\times$ ____) + ____ $= 11111$

**8** Fill in the gaps in the following pattern.

$(1 \times 8) + 1 = 9$

$(12 \times 8) +$ ____ $= 98$

(_____ $\times 8) + 3 = 987$

(_____ $\times$ ____) + ____ = _____

**9** The table on the next page shows the number of rabbits in a warren at the same time each year, for the years listed.

The rabbit colony began in 1995 with one pair of rabbits. On average, each pair of rabbits has six baby rabbits (kittens). Assume that equal numbers of male and female kittens are born and they have their first litter at the age of 1 year.

**a** Complete the table.

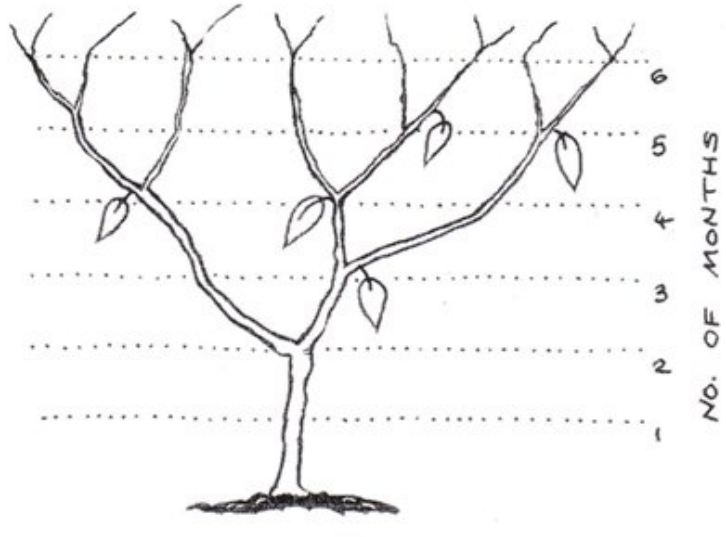
| Year | 1995 | 1996 | 1997 | 1998 | 1999 | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| No. of kittens | | 6 | 24 | 96 | | | | | | | | |
| No. of rabbits | 2 | 8 | 32 | 128 | | | | | | | | |

**b** Without doing the calculations for the years 2007–09, how many rabbits do you expect to be in the warren in the year 2010?

**c** Without doing the calculations for the years 2007–08, how many kittens do you expect to be born in 2009?

**10** Fibonacci numbers form the pattern:
1, 1, 2, 3, 5, ....
Some plants show the Fibonacci numbers in the number of branches that shoot each month. This is shown in the diagram.

**a** Write the number of branches each month in the table.




| Month | No. of branches |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |

**b** Describe the pattern you see in the number of branches. What will the number of branches be in Month 7?

**c** Use the pattern to find out how many branches you would expect after 2 years (24 months).

**d** If we find the ratio of the first two numbers in the Fibonacci series, we get: $\frac{1}{1}=1$

If we continue finding the ratios of consecutive numbers in the Fibonacci series, we get:

$\frac{2}{1}=2, \frac{3}{2}=1.5, \frac{5}{3}=1.666, \frac{8}{5}=1.6, \frac{13}{8}=1.625...$ approximately.

Use your calculator to show that the ratios are getting closer and closer to approximately 1.618. This number is called the Golden Ratio or Golden Number, and is considered to produce shapes pleasing to the eye.

# Answers

## 1 Fun with units

**1** **a** 3 000 000 000 metres or $3 \times 10^9$ metres
**b** 6 000 000 grams or $6 \times 10^6$ grams
**c** 0.005 metres or $5 \times 10^{-3}$ metres
**2** **a** 3 000 000 000 g, 5 000 000 g, 4 000 g, 3 005 004 000 g
**b** 3 000 000 000 m, 5 000 000 m, 4 000 m, 3 005 004 000 m
**3** megalitre **4** Fly's wing is heavier **5** Too fast; more like 1 mm per day. **6** No; too large.
**7** Both are the same.
**8** **a** 1 000 000 000 000 pins or $10^{12}$ pins
**b** 1 000 whales or $10^3$ whales
**c** 0.1 mates or $10^{-1}$ mates
**d** 0.01 pedes or $10^{-2}$ pedes
**e** 0.000 000 001 goats or $10^{-9}$ goats
**f** 0.000 000 000 001 doors or $10^{-12}$ doors!
**9** **a** centimetre **b** kilogram **c** hectare **d** tonne **e** micron **f** litre
**10** X

## 2 Reach for the stars

**1** 9 461 000 000 000 000 metres or $9.461 \times 10^{15}$ metres
**2** Distances are much too large.
**3** 26 000 light years away
**4** 100 000 light years across **5** 27 years
**6** Tennis match (500 seconds = 8.33 minutes)
**7** ~ 149 597 871 km
**8** **a** 1 light year **b** 1 light year = 63 241.077 AU approximately
**9** 12 994 000°C
**10** Sun costs $\$50 \times 10^{30}$ and Earth costs $\$100 \times 10^{25}$ so Sun costs more.
**11** 1257 $km^2$
**12** 266 877.34 AU
**13** 30 842 860 000 000
**14** 170.77 years
**15** **a** 14 hours and 1 minute and 4 seconds
**b** 31 minutes and 19 seconds

## 3 Patterns in Science

**1** 13, 16, 19 **2** –3, –7, –11
**3** 16, 32, 64 **4** 16, 25, 36
**5** 10, 15, 21 **6** 27, 8, 1
**7** 3; 123; 1234; 9; 5 **8** 2; 123; 1234; 8; 4; 9876
**9** **a** Kittens: 384, 1536, 6144, 24 576, 98 304, 393 216, 1 572 864, 6 291 456; Rabbits: 512, 2048, 8192, 32 768, 131 072, 524 288, 2 097 152, 8 388 608
**b** approximately 2 147 483 000
**c** approximately 402 653 160
**10** **a** 1, 1, 2, 3, 5, 8
**b** Each number is the sum of the previous 2 numbers; 13.
**c** 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10 946, 17 711, 28 657, 46 368
**d** Teacher to check.

## 4 How hot is that?

**1** 23° **2** 12° **3** 38°C **4** 14°C
**5** 116°F **6** A: –2°C and –5°C
**7** No, likely to be concentrated sea water.
**8** 36.4°C to 37.6°C **9** 37.4°C to 37.7°C
**10** 36.9°C to 38.1°C **11** Yes, it is 27°C
**12** **a** Teacher to check
**b** 75°C; –15°C; above 60°C; below 5°C; below 5°C
**13** **a** 90°C **b** 10°C **c** No, because it was not heated to above 75°C.

## 5 Movement

**1** 118 kph **2** 1000 s or 16.67 min **3** $\frac{1}{9}$ second
**4** **a** 58 km **b** 464 km **c** 11 hours
**5** **a** 2.25 h **b** 3 h **c** 45 minutes
**6** **a** 1382 km **b** ~ 92.13 km per hour
**c** 110/13.82 = ~7.96 L per 100 km
**7** **a** 800 km **b** 16 hours
**8** 13.5 m **9** The second car.
**10** **a** speeding up **b** 3 m $s^{-1}$ **c** 3 m $s^{-2}$
**11** **a** C **b** B, C **c** A **d** D **e** C
**12** **a** A **b** C

## 6 Blood and skin

**1** **a** 6 900 000 000 or $6.9 \times 10^9$
**b** 165 600 000 000 or $1.656 \times 10^{11}$
**c** 60 444 000 000 000 or $6.0444 \times 10^{13}$
**d** 4 231 080 000 000 000 or $4.23108 \times 10^{15}$
**2** **a** 1250 **b** 80 000 **c** Yes
**3** 40 **4** 9.1 m **5** 68.4 km **6** 80 000 **7** 0.43̇ s
**8** **a** e.g. 44 kg **b** e.g. 4 L **c** 66 kg
**9** 135 g
**10** 382500 km
**11** $1.9025 \times 10^{-7}$ g
**12** **a** 9870 L **b** 9.4% **c** 470 000 L
**13** ~ 47 times)
**14** $x = 2$, $y = 12$, $z = 0.5$

## 7 Heads and tails

**1** 14 **2** 32 **3** 9000 **4** 100 000 **5** 3
**6** ~25 kg; ~39.1 g
**7** **a** 720 **b** 11 520
**8** **a** 9 **b** 3333
**9** 21 cm to 35 cm **10** 6.97 kg
**11** 59.5 mm to 115.5 mm
**12** (In table order)
Total length: 46, 80, 152, 272, 58, 49, 250, 1600, 360
Tail length as % of total length: 24, 68.8, 21.1, 39.3, 44.8, 22.5, 60, 18.8, 16.7
**13** **a** platypus, kookaburra, dingo/striped possum, lyrebird, diprotodon, red kangaroo, lace monitor, humpback whale
**b** striped possum, lyrebird/kookaburra, platypus, lace monitor, dingo, red kangaroo, diprotodon, humpback whale
**c** lyrebird
**d** diprotodon

## 8 Body parts

**1** 180 min **2** 5 **3** 1400 g **4** ~2.9%
**5** 18 000 000 **6** $\frac{18}{39}$ kg or 28 g
**7** **a** 7.5 m **b** 4.125 km **c** ~134 students
**8** 14.4 m
**9** **a** ~11 500 (11 520) **b** ~4.2 million (4 204 800)
**10** 0.0169 s
**11** **a** 45.83 m $s^{-1}$ **b** 0.065 s; unlikely